ON THE GALACTOCENTRIC DISTANCE OF THE SUN AND ITS ROTATION VELOCITY

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Abstract

Two methods to evaluate R_0 and ω_0, using radial velocities and proper motions of cepheids, are introduced: (i). R_0 can be derived by evaluating $R_0 \Delta\omega$ against ΔR for various values of R_0 under the condition that $R_0 \Delta\omega = 0$ at $R = R_0$, and (ii). ω_0 can be derived by evaluating ω against ΔR, and the ratio of $R_0 \Delta\omega$ to $\Delta\omega$ gives R_0.

Using Irwin's data on cepheids, above methods yield $R_0 = 8.6$ kpc, and $\omega_0 = 29.0 \pm 9.0$ km sec^{-1} kpc^{-1}. These values are favorable for excluding the inconsistency existing between the available values of $R_0\omega_0$ and V_0.

1. Introduction.

As is pointed out by Weaver and Morgan (1956) and by Weaver (1956, 1959), there exists an inconsistency among available values of galactic constants R_0, ω_0, and V_0. The value of galactocentric distance of the sun, $R_0 = 8.2$ kpc, is a mean between Baade's (1954) value 8.16 kpc derived from the distribution of RR Lyrae variable in the region of the galactic centre and van de Hulst, Muller and Oort's (1954) value 8.26 kpc derived from 21 cm observation.

For the angular velocity of galactic rotation, van de Hulst, Muller and Oort (1954) found $\omega_0 = 26.4$ km sec^{-1} kpc^{-1} from the difference of A and B. Later, Weaver and Morgan (1956) used the proper motions of cepheids to find $\omega_0 = 23.2$ km sec^{-1} kpc^{-1}. These values of ω_0, combined with $R_0 = 8.2$ kpc, give the circular velocity $V_0 = 216$ km sec^{-1} and 190 km sec^{-1}, respectively.

Direct determinations of V_0, on the other hand, show fairly larger values. Mayall (1946) obtained $V_0 = 200$ km sec^{-1} from radial velocities of globular clusters. By determining the solar motion with respect to the local group of extragalactic nebulae he concluded $V_0 = 300$ km sec^{-1}, and Humason and Wahlquist (1955) obtained $V_0 = 292$ km sec^{-1}. Fricke (1948, 1949) used the high velocity stars to compute $V_0 = 276$ km sec^{-1}.

This incosistency in V_0 suggests the underestiamations of current values either of R_0 and ω_0, or both. The independent attacks to investigate them are strongly needed.

In this paper two methods using the kinematical data of cepheids are introduced. One gives R_0 from radial velocities, and the other gives ω_0 and R_0 from proper motions. Each of these methods does not need to assume any other galactic constants. This is the essential difference from the other methods hitherto employed.

By means of Irwin's data on cepheids, these methods are tried, and give preferable results.

2. Fundamental Equations and Data.

We adopt customary assumptions for the galactic rotation, *i.e.* (i). that the galactic system is in a steady state, (ii). that the stars move in circular orbits around the galactic centre, and (iii). that any star's z-distance is sufficiently small, and hence we can put $\omega(R, z)=\omega(R, 0)\equiv\omega(R)$.

Then the fundamental equations for the galactic rotation are, for radial velocity

$$v_r = R_0 \Delta\omega(R) \sin\lambda \cos b, \tag{1}$$

and for proper motion

$$\kappa\mu_l = \frac{R_0\Delta\omega(R)}{r}\cos\lambda \sec b - \omega(R), \tag{2}$$

$$\kappa\mu_b = -\frac{R_0\Delta\omega(R)}{r}\sin\lambda \sin b, \tag{3}$$

here, symbols are also customary ones.

Dr. J. B. Irwin kindly provided his data on 145 cepheids made at Cape and Radcliff in 1955. These data contain the distances r from the sun, which are computed by his period-luminosity and period-colour relations (Irwin, 1958).

For radial velocities the data by Stibbs (1955), and for proper motions the data by Morgan (Weaver and Morgan, 1956) are adopted, respectively. The former includes 54 and the latter includes 49 stars which are common with Irwin's stars. Radial velocities are corrected for the solar motion $V_{\odot}=19.5$ km sec^{-1} towards $L_{\odot}=23°.5$ and $B_{\odot}=+21°.6$.

These data of distances, radial velocities and proper motions are considered to be most reliable ones obtainable at present, though Dr. Irwin remarked to terat his data with care because of their provisionality.

3. R_0 from Radial Velocities.

Eq. (1) is rewritten as

$$R_0\Delta\omega(R) = \frac{v_r}{\sin\lambda\cos b} \equiv u. \tag{4}$$

The relation between u and R can be obtained if we assume a value of R_0. Let $R_0{}^*$ be an assumed value. At $R=R_0$, $\omega=\omega_0$, hence $u=0$. Therefore, the $u\sim R$ relation should yield $u=0$ at $R=R_0{}^*$ if we assume correctly $R_0{}^*=R_0$.

In Fig. 1, C and C* are true and assumed centres of galaxy, respectively,

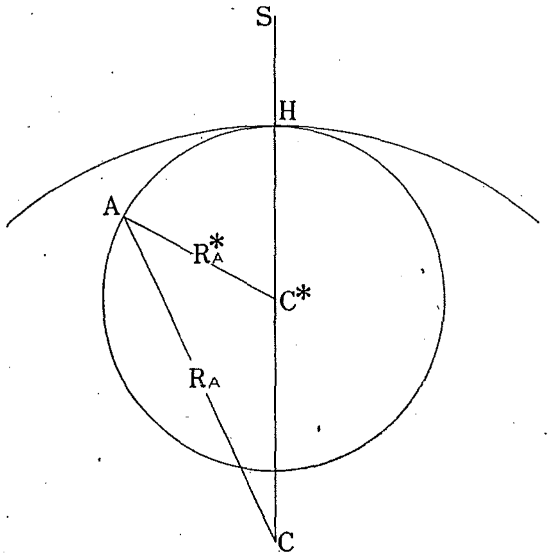

Fig. 1.

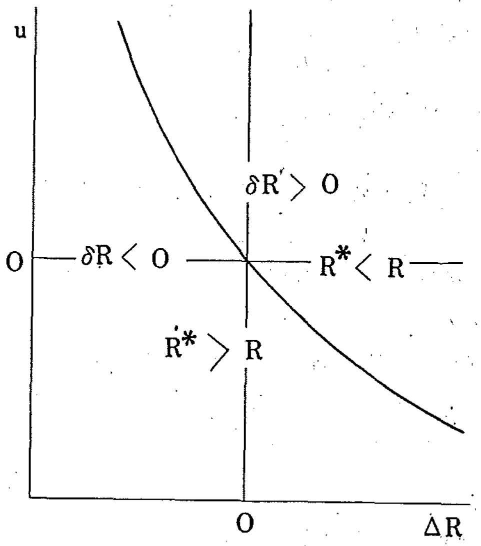

Fig. 2.

then the galactocentric distances of the sun S are R_0=CS and R_0^*=C*S, respectively. If $R_0^*<R_0$, as shown in the figure, the computed galactocentric distance R_A=C*A of a star A is smaller than the true distance R_A=CA. As is stated dynamically, the angular velocity ω of galactic rotation seems to decrease monotonously with R in the range $R-R_0>-2$ kpc. Therefore, the angular velocity of A is larger than that of H, which situates in the direction towards the galactic centre and has common computed distance R_A^*=C*H. Then, if we plot u against R^*, stars scatter along a curve which runs in the upper region than the true $u\sim R$ curve. Analogously, if R_0^* is overestimated, stars scatter along a curve lower than the true one. Namely, (Fig. 2)

$$\delta R_0>0 \quad \text{if} \quad R_0^*<R_0$$
$$<0 \qquad\qquad >R_0,$$

where

$$\delta R_0=R_{u=0}-R_0^*. \tag{5}$$

Therefore, when we adopt larger values of R_0^*, successively, δR_0 may decrease monotonously, giving true value $R_0^*=R_0$ at $\delta R_0=0$. (Really, in Fig. 1*b* of the Weaver's paper (1955), we can recognize $\delta R_0>0$.)

Among 54 stars mentioned in the last section, 44 stars satisfy a condition $|\sin\lambda\cos b|>0.5$. For these stars, regression curves of u on $\Delta R=R-R_0^*$ and of ΔR on u are constructed for $R^*=6.8\sim9.6$ kpc. Weights are assigned for the distance r to be 1, 2, and 4 corresponding to the notes *very doubtful*, *doubtful*, and other, respectively, in Irwin's data. In this range δR_0 decreases 0.014 kpc per kpc. At $R_0^*=8.20$ kpc the empirical relations are expressed by (Fig. 3)

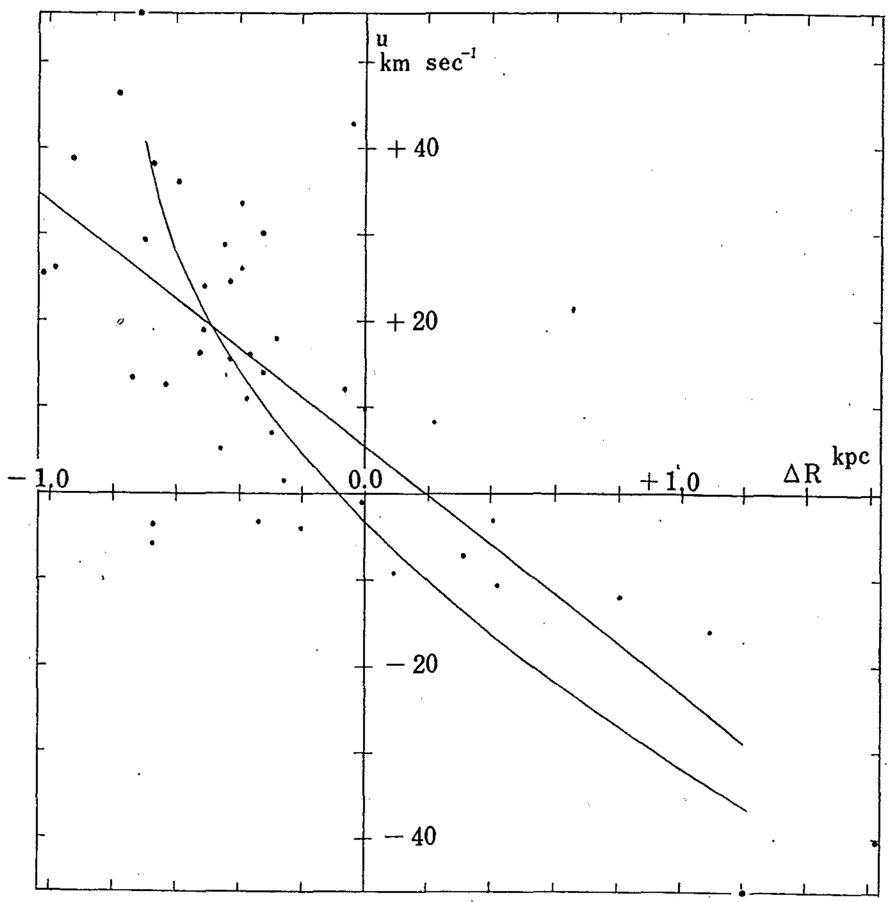

Fig. 3. $u-\Delta R$ Relation for $R_0{}^*=8.2$ kpc.

$$u = 5.63-28.4(\Delta R)-0.31(\Delta R)^2, \tag{6}$$

$$\Delta R = -0.087-0.0259\,u+0.00027\,u^2. \tag{7}$$

Assuming the errors in v_r to be twice those in R, we have $\delta R=0.006$ kpc at $R_0{}^* = 8.20$ kpc. Hence, we obtain

$$R_0=8.20+\frac{0.006}{0.014}=8.6\text{ kpc}.$$

This value cannot be considered to be definite, since the obtained values of δR_0 and its increment for R are both too small. Though this will be discussed again in the following section, we can recognize that the current value $R_0{}^*$ $= 8.2$ kpc might be a underestimated value.

Baade (1955), in his derivation of R_0, used RR Lyrae stars in the region of the galactic nucleus, but it is very difficult to estimate the interstellar extinction in that region. Van de Hulst, Muller and Oort (1954) had to use the galactic constant A in their evaluation. On the contrary, present method is purely kinematical and does not need to assume any constant, and only requires accuracy in r and v_r for much nearer stars than the galactic nucleus.

4. ω_0 from Proper Motions.

For small value of r, we can expand

$$\Delta R = R - R_0 = -r\cos\lambda + \frac{1}{2}\frac{r^2}{R_0}\sin^2\lambda. \tag{8}$$

If R_0 is estimated incorrectly, *i.e.* $\Delta R_0 = R_0 - R_0^* \neq 0$,

$$\Delta(\Delta R) = -\frac{1}{2}\left(\frac{r}{R_0}\right)^2 \sin^2\lambda\,\Delta R_0. \tag{9}$$

For $R_0 = 8.2$ and $\Delta R_0 = 1$ kpc, we have

$$\begin{array}{lll} \Delta(\Delta R) < 0.007\,\text{kpc} & \text{for} & r = 1\,\text{kpc}, \\ \quad < 0.03 & & \quad = 2 \quad . \end{array}$$

These $\Delta(\Delta R)$ are in the range of error in distance estimation. Hence, ΔR is rather insensitive for the change in R_0^*. This insensitivity is the foible of the method introduced in the last section, since δR_0 and its increment are too small. However, we can utilize this feature reversely. We have enough accurate values of ΔR even though the assumed value of R_0^* be uncertain to some extent. Namely, the $u \sim \Delta R$ relation holds enough accurately for the range $|R_0 - R_0^*| < 2$ kpc.

The relation between proper motion in declination and those in galactic longitude and in latitude is

$$\mu_\delta = \mu_l \cos b \sin\varphi + \mu_b \cos\varphi + \Delta n \cos\alpha, \tag{10}$$

where φ is parallactic angle subtended to the equatorial and galactic poles, and n, as ordinary use, is precession in declination.

Substituting (2) and (3) into (10), we have

$$\omega(\Delta R)\cos b \sin\varphi = \kappa\Delta n \cos\alpha - \kappa\mu_\delta$$
$$+ \frac{R_0 \Delta\omega(\Delta R)}{r}(\cos\lambda\sin\varphi - \sin\lambda\sin b\cos\varphi). \tag{11}$$

Though this equation is same as eq. (5) in Weaver and Morgan's (1956) paper substantially, it must be remarked that, in the present case, ω and $\Delta\omega = \omega - \omega_0$ are treated as the functions of ΔR, respectively, instead of R itself.

The right side of (11) can be evaluated numerically, since we have $R_0\Delta\omega(\Delta R) = u(\Delta R)$. Then, we can obtain the relation between ω and ΔR, which gives ω_0 at $\Delta R = 0$. Moreover, it gives $\Delta\omega(\Delta R)$ from which R_0 can be evaluated immediately by comparing with $u(\Delta R)$.

In their derivations of ω_0, van de Hulst, Muller and Oort (1954) used A and B, and Weaver and Morgan (1956) used R_0. On the contrary, the above method does not need any assumption on the other galactic constants, and only requires ample data of proper motions distributed equally along the galactic equator.

The 39 stars, for which both proper motions and distances are available and for which $|\cos b \sin\varphi| > 0.55$, are computed their ω's by (11), adopting (6) for $u \sim \Delta R$ relation and $\Delta p = +0.''80$ per century (Miyadi, 1958) for the correction to Newcomb's precession constant. The distribution of ω against

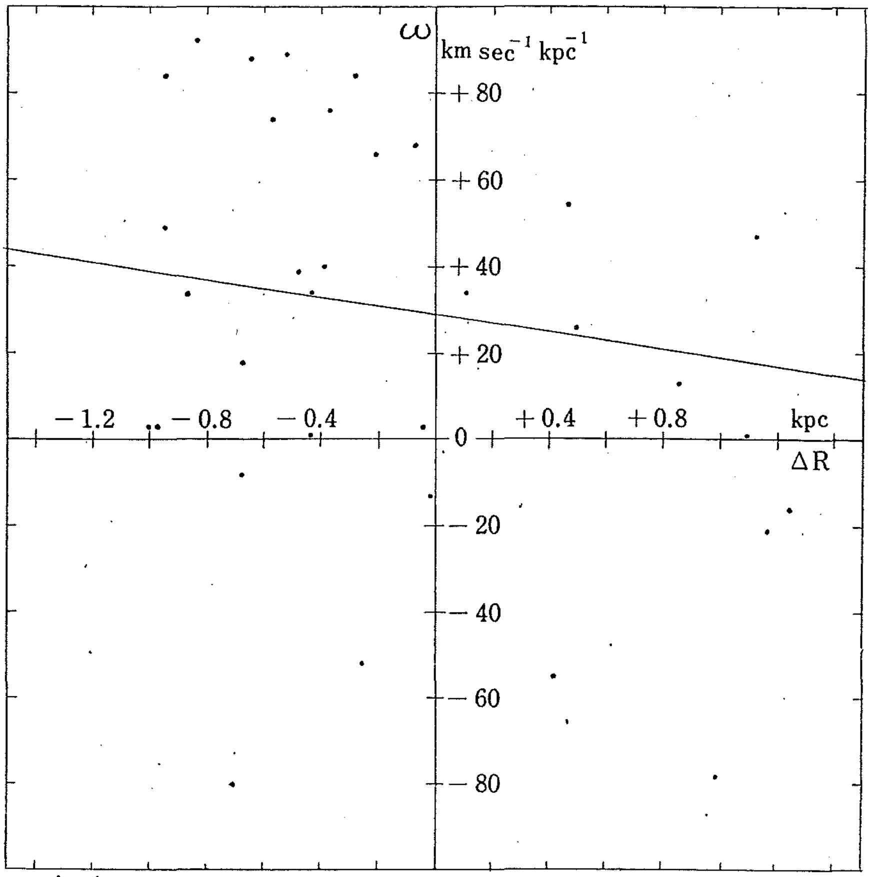

Fig. 4. $\omega-\varDelta R$ Relation

$\varDelta R$ is shown in Fig. 4, which gives by the method of least squares:

$$\omega \doteq 29.0 - 9.9\varDelta R \ \text{km sec}^{-1}\ \text{kpc}^{-1}. \qquad (12)$$
$$\pm 9.0 \pm 9.7$$

In this computation weight is not taken into account tentatively.
For $\varDelta R=0$, we obtain

$$\omega_0 = 29.0 \pm 9.0 \ \text{km sec}^{-1}\ \text{kpc}^{-1}.$$

In order to get a plausible value of R_0 the coefficient of $\varDelta R$ in (12) has to be reduced to about one third or fourth. The number of data is too small to obtain R_0, but this inplausibility of the value of the coefficient of $\varDelta R$ affects little the value of ω at $\varDelta R=0$.

The other constants are evaluated for convenience to the comparison as follows:

$$A = +14.2 \ \text{km sec}^{-1}\ \text{kpc}^{-1},$$
$$B = -14.8 \ \text{km sec}^{-1}\ \text{kpc}^{-1},$$

and

$$V_0 = 238 \ \text{km sec}^{-1} \quad \text{for} \quad R_0 = 8.2\,\text{kpc},$$
$$V_0 = 250 \ \text{km sec}^{-1} \quad \text{for} \quad R_0 = 8.6\,\text{kpc}.$$

5. Conclusions.

Two absolute methods to evaluate the galactic constants, using the fundamental equations of galactic rotation by the kinematical data of cepheids, are introduced, both of which, contrary to the other methods, do not necessitate any assumption on the values of the other galactic constants;

(i). R_0 from radial velocities: Compute $R_0\Delta\omega$ against $\Delta R_0=R-R_0$ for various values of R_0, then R_0 can be fixed by the condition that $R_0\Delta\omega$ should be zero at $R=R_0$.

(ii). ω_0 and R_0 from proper motions: Compute ω against ΔR, then ω_0 can be fixed at $\Delta R=0$. The ratio of $R_0\Delta\omega$ to $\Delta\omega$ gives R_0.

Using the data of r by Irwin, v_r by Stibbs, and μ by Morgan, following values are obtained:

$R_0=8.6\,\mathrm{kpc}$ from radial velocities,

$\omega_0=29.0\pm9.0\,\mathrm{km\ sec^{-1}\ kpc^{-1}}$ from proper motions.

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