

NUMERICAL VALUES OF THE INTEGRALS OF

$(2/\sqrt{\pi})\exp(-x^2)$, $(2/\sqrt{\pi})x^2\exp(-x^2)$, AND $(2/\sqrt{\pi})x^4\exp(-x^2)$

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Abstract

Tables for the numerical figures of the functions $\frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$, $\frac{2}{\sqrt{\pi}} \int_0^x x^2 e^{-x^2} dx$, and $\frac{2}{\sqrt{\pi}} \int_0^x x^4 e^{-x^2} dx$ are constructed for the values of the argument x from 0.0 to 8.0, respectively.

1. Introduction

In this paper are given the numerical figures of functions $(2/\sqrt{\pi})\exp(-x^2)$, $(2/\sqrt{\pi})x^2\exp(-x^2)$, and $(2/\sqrt{\pi})x^4\exp(-x^2)$, which frequently appear in the theory of errors, and also in statistical mechanics and sometimes in stellar dynamics.

Most tables now existing (e. g., E. Jahnke u. F. Emde, 1928, and K. Hayashi, 1941) for the error function defined as the integral of the above first function, give practically insignificant figures for larger values of the argument; for instance, a certain seven decimal table of the error function gives a value, 0.9999998 for $x=4.0$. However, some problems, e. g., appearing in a certain field of stellar dynamics, need several significant figures after a series of 9's. In the present tables, a special attention is paid to keep the figures significant even for larger values of the argument.

2. Methods of Evaluation

We shall define the error function and the other similar functions as

$$\begin{aligned} \Phi_0(x) &= \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx, & \Phi_0'(x) &= \frac{2}{\sqrt{\pi}} e^{-x^2}, \\ \Phi_2(x) &= \frac{2}{\sqrt{\pi}} \int_0^x x^2 e^{-x^2} dx, & \text{and} & & \Phi_4(x) &= \frac{2}{\sqrt{\pi}} \int_0^x x^4 e^{-x^2} dx. \end{aligned} \quad (1)$$

It is easily seen that

$$\Phi_2(x) = \frac{1}{2} \left\{ \Phi_0(x) - x\Phi_0'(x) \right\}, \quad \Phi_4(x) = \frac{3}{2} \left\{ \Phi_2(x) - \frac{1}{3} x^3 \Phi_0'(x) \right\}, \quad (2)$$

and that

$$\Phi_0(x) \rightarrow 1, \quad \Phi_2(x) \rightarrow \frac{1}{2}, \quad \text{and} \quad \Phi_4(x) \rightarrow \frac{3}{4}, \quad \text{for } x \rightarrow \infty, \quad (3)$$

respectively.

The functions $\Phi_0'(x)$, $\Phi_0(x)$, $\Phi_2(x)$, and $\Phi_4(x)$ are computed for the values of x ranging from 0.0 to 8.0. Methods of the numerical evaluation are:

(i) For $0.00 \leq x \leq 1.00$, $\Phi_4(x)$ is computed through the power series,

$$\Phi_4(x) = \frac{2x^5}{\sqrt{\pi}} \left[\frac{1}{5} - \frac{x^2}{1!7} + \frac{x^4}{2!9} - \frac{x^6}{3!11} + \frac{x^8}{4!13} - (\text{down to the term of } x^{16}) \right], \quad (4)$$

and the eqs. (2) are employed for the other two functions.

(ii) For $1.1 \leq x \leq 2.8$, the values of the error function, $\Phi_0(x)$, are taken from the Y. Watanabe's seven figure table (1943); and these of the other two functions are obtained also with the aid of the eqs. (2).

(iii) For $2.9 \leq x \leq 8.0$, the asymptotic expansion is employed for the error function:

$$\Phi_0(x) \sim 1 - \frac{e^{-x^2}}{x\sqrt{\pi}} \left[1 - \frac{1}{2x^2} + \frac{1 \cdot 3}{(2x^2)^2} - \frac{1 \cdot 3 \cdot 5}{(2x^2)^3} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{(2x^2)^4} - \dots \right]. \quad (5)$$

Accuracy of this expansion is not very high for the smaller values of x , say from 2.9 to 3.1; and the resulted functional values will include errors about ± 5 in the unit of final decimal of the tabulated figures. However, the errors affect very little to the values of $\Phi_2(x)$ and $\Phi_4(x)$ obtained through the eqs. (2). For the larger values of x , the expansion (5) yields amply accurate results.

3. Construction and Use of the Tables

The Table 1 gives the six decimal figures of $\Phi_0'(x)$, $\Phi_0(x)$, $\Phi_2(x)$, and $\Phi_4(x)$, and their first differences for every 0.01 values of x ranging from 0.00 to 1.00. To interpolate these functions, it is enough only to take the differences down to the second into account for almost all over the range of x , except for the cases of $\Phi_2(x)$ and $\Phi_4(x)$; in which the third and fourth differences will be needed for $0.00 \leq x \leq 0.30$.

In the Table 2 are given the values of $1 - \Phi_0(x)$, $\frac{1}{2} - \Phi_2(x)$, $\frac{3}{4} - \Phi_4(x)$, and their logarithms, with the natural values of $\Phi_0'(x)$, for every 0.1 values of x from 0.8 to 8.0. The first differences of the logarithmic values of the former three functions are given. For these three, the interpolation should be made with respect to the logarithmic values, *not* to the natural ones, by employing the first *and* second differences.

The errors in the tabulated values do not exceed ± 2 in the unit of the final decimal, except for the values noted in (iii), Section 2.

References

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TABLE 1. VALUES OF $\Phi_0'(x)$, $\Phi_0(x)$, $\Phi_2(x)$ AND $\Phi_4(x)$
FOR EVERY 0.01 VALUE OF x FROM 0.00 TO 1.00.

x	$\Phi_0'(x)$	$\Phi_0(x)$	$\Phi_2(x)$	$\Phi_4(x)$
0.00	1.128379	0.000000	0.00000	0.00000
.01	1.128266 - 113	.0112834 112834	3.76104 $\cdot 10^{-7}$ 376104	2.25660 $\cdot 10^{-11}$ 225660
.02	1.127928 - 338	.0225646 112812	3.00829 $\cdot 10^{-6}$ 263219	7.21956 $\cdot 10^{-10}$ 689390
.03	1.127364 - 564	.0338412 112766	1.01499 $\cdot 10^{-5}$ 71416	5.48040 $\cdot 10^{-9}$ 475844
.04	1.126575 - 789	.0451111 112699	2.40490 138991	2.30828 $\cdot 10^{-8}$ 176024
.05	1.125562 -1013	.0563720 112609	4.69453 228963	7.03979 $\cdot 10^{-8}$ 473151
.06	1.124324 -1238	.0676216 112496	8.10680 $\cdot 10^{-5}$ 341227	1.75035 $\cdot 10^{-7}$ 104637
.07	1.122864 -1460	.0788577 112361	1.28633 $\cdot 10^{-4}$ 47565	3.77968 202933
.08	1.121181 -1683	.0900781 112204	1.91839 63206	7.36122 $\cdot 10^{-7}$ 35814
.09	1.119276 -1905	.101281 11203	2.72867 81028	1.32491 $\cdot 10^{-6}$ 58879
	-2124	11182	101011	91579
0.10	1.117152	.112463	3.73878	2.24070
.11	1.114808 -2344	.123623 11160	4.97005 123127	3.60324 136254
.12	1.112247 -2561	.134758 11135	6.44360 147355	5.55809 195485
.13	1.109470 -2777	.145867 11109	8.18021 $\cdot 10^{-4}$ 173661	8.27869 $\cdot 10^{-6}$ 272060
.14	1.106478 -2992	.156947 11080	1.02004 $\cdot 10^{-3}$ 20202	1.19687 $\cdot 10^{-5}$ 36900
.15	1.103274 -3204	.167996 11049	1.25243 22239	1.68642 48955
.16	1.099859 -3415	.179012 11016	1.51716 26473	2.32354 63712
.17	1.096236 -3623	.189992 10980	1.81619 29903	3.13886 81532
.18	1.092406 -3830	.200936 10944	2.15142 33523	4.16684 102798
.19	1.088371 -4035	.211840 10904	2.52469 37327	5.44587 127903
	-4236	10863	41313	157270
0.20	1.084135	.222703	2.93782	7.01847
.21	1.079699 -4436	.233522 10819	3.39257 45475	8.93142 $\cdot 10^{-5}$ 191295
.22	1.075066 -4633	.244296 10774	3.89067 49810	1.12359 $\cdot 10^{-4}$ 23045
.23	1.070239 -4827	.255023 10727	4.43378 54311	1.39876 27517
.24	1.065221 -5018	.265700 10677	5.02352 58974	1.72467 32591
.25	1.060014 -5207	.276326 10626	5.66143 63791	2.10783 38316
.26	1.054622 -5392	.286900 10574	6.34903 68760	2.55521 44738
.27	1.049047 -5575	.297418 10518	7.08775 73872	3.07427 51906
.28	1.043293 -5754	.307880 10462	7.87899 79124	3.67295 59868
.29	1.037363 -5930	.318284 10404	8.72406 84507	4.35970 68675
	-6102	10343	90018	78372
0.30	1.031261	.328627	9.62424 $\cdot 10^{-3}$	5.14342
.31	1.024989 -6272	.338908 10281	1.05807 $\cdot 10^{-2}$ 9565	6.03356 89014
.32	1.018552 -6437	.349126 10218	1.15946 10139	7.04001 100645
.33	1.011953 -6599	.359279 10153	1.26671 10725	8.17316 113315
.34	1.005196 -6757	.369365 10086	1.37990 11319	9.44391 $\cdot 10^{-4}$ 127075
.35	0.998284 -6912	.379382 10017	1.49914 11924	1.08636 $\cdot 10^{-3}$ 14197
.36	.991221 -7063	.389330 9948	1.62451 12537	1.24440 15804
.37	.984011 -7210	.399206 9876	1.75609 13158	1.41975 17535
.38	.976659 -7352	.409009 9803	1.89395 13786	1.61367 19392
.39	.969168 -7491	.418739 9730	2.03817 14422	1.82749 21382
	-7627	9653	15062	23507
0.40	.961541	.428392	2.18879	2.06256
.41	.953784 -7757	.437969 9577	2.34588 15709	2.32027 25771
.42	.945901 -7883	.447468 9499	2.50947 16359	2.60207 28180
.43	.937895 -8006	.456887 9419	2.67960 17013	2.90944 30737
.44	.929770 -8125	.466225 9338	2.85631 17671	3.24388 33444
.45	.921532 -8238	.475482 9257	3.03962 18331	3.60693 36305
.46	.913184 -8348	.484655 9173	3.22954 18992	4.00018 39325
.47	.904731 -8453	.493745 9090	3.42608 19654	4.42522 42504
.48	.896177 -8554	.502750 9005	3.62924 20316	4.88368 45846
.49	.887526 -8651	.511668 8918	3.83903 20979	5.37723 49355
	-8743	8832	21640	53030
0.50	0.878783	0.520500	4.05543 $\cdot 10^{-2}$	5.90753 $\cdot 10^{-3}$

TABLE 1. (Continued)

x	$\Phi_0'(x)$	$\Phi_0(x)$	$\Phi_2(x)$	$\Phi_4(x)$
0.50	0.878783	0.520500	0.0405543	0.00590753
.51	.869951 -8832	.529243 8743	.0427842 22299	.00647628 56875
.52	.861037 -8914	.537899 8656	.0450797 22955	.00708518 60890
.53	.852043 -8994	.546464 8565	.0474405 23608	.00773597 65079
.54	.842975 -9068	.554939 8475	.0498663 24258	.00843037 69440
.55	.833837 -9138	.563323 8384	.0523566 24903	.00917012 73975
.56	.824632 -9205	.571616 8293	.0549108 25542	.00995697 78685
.57	.815366 -9266	.579816 8200	.0575285 26177	.0107927 8357
.58	.806043 -9323	.587923 8107	.0602089 26804	.0116790 8863
.59	.796667 -9376	.595936 8013	.0629513 27424	.0126176 9386
0.60	.787243	.603856	.0657550	.0136102
.61	.777775 -9468	.611681 7825	.0686191 28641	.0146587 10485
.62	.768267 -9508	.619412 7731	.0715430 29239	.0157647 11060
.63	.758724 -9543	.627046 7634	.0745253 29823	.0169297 11650
.64	.749148 -9576	.634586 7540	.0775653 30400	.0181556 12259
.65	.739547 -9601	.642029 7443	.0806619 30966	.0194439 12883
.66	.729922 -9625	.649377 7348	.0838141 31522	.0207964 13525
.67	.720278 -9644	.656628 7251	.0870206 32065	.0222145 14181
.68	.710620 -9658	.663782 7154	.0902804 32598	.0236998 14853
.69	.700950 -9670	.670840 7058	.0935920 33116	.0252537 15539
0.70	.691275	.677801	.0969544	.0268779
.71	.681596 -9679	.684665 6864	.100366 3412	.0285737 16958
.72	.671919 -9677	.691433 6768	.103826 3460	.0303425 17688
.73	.662246 -9673	.698104 6671	.107332 3506	.0321856 18431
.74	.652582 -9664	.704678 6574	.110884 3552	.0341042 19186
.75	.642931 -9651	.711156 6478	.114479 3595	.0360997 19955
.76	.633296 -9635	.717537 6381	.118116 3637	.0381732 20735
.77	.623680 -9616	.723822 6285	.121794 3678	.0403257 21525
.78	.614088 -9592	.730011 6189	.125511 3717	.0425583 22326
.79	.604522 -9566	.736103 6092	.129266 3755	.0448721 23138
0.80	.594986	.742101	.133056	.0472679
.81	.585483 -9503	.748003 5902	.136881 3825	.0497465 24786
.82	.576017 -9466	.753811 5808	.140738 3857	.0523087 25622
.83	.566591 -9426	.759524 5713	.144627 3889	.0549553 26466
.84	.557207 -9384	.765142 5618	.148544 3917	.0576868 27315
.85	.547870 -9337	.770668 5526	.152489 3945	.0605038 28170
.86	.538581 -9289	.776100 5432	.156460 3971	.0634068 29030
.87	.529343 -9238	.781440 5340	.160455 3995	.0663960 29892
.88	.520161 -9182	.786687 5247	.164473 4018	.0694720 30760
.89	.511035 -9126	.791843 5156	.168511 4038	.0726348 31628
0.90	.501969	.796908	.172568	.0758846
.91	.492965 -9004	.801882 4974	.176642 4074	.0792216 33370
.92	.484025 -8930	.806768 4886	.180732 4090	.0826456 34240
.93	.475153 -8872	.811560 4792	.184835 4103	.0861566 35110
.94	.466350 -8803	.816270 4710	.188950 4115	.0897542 35976
.95	.457619 -8731	.820890 4620	.193076 4126	.0934383 36841
.96	.448962 -8657	.825423 4533	.197210 4134	.0972086 37703
.97	.440380 -8582	.829869 4446	.201351 4141	.101064 3855
.98	.431876 -8504	.834231 4362	.205496 4145	.105005 3941
.99	.423451 -8425	.838507 4276	.209645 4149	.109031 4026
1.00	0.415107	0.842699	0.213796	0.113140

TABLE 2. VALUES OF $\phi_0'(x)$, $1-\phi_0(x)$, $\frac{1}{2}-\phi_2(x)$ AND $\frac{3}{4}-\phi_4(x)$
FOR EVERY 0.1 VALUE OF x FROM 0.8 TO 8.0

x	$\phi_0'(x)$ *	$1-\phi_0(x)$	$\log(1-\phi_0(x))$	$\frac{1}{2}-\phi_2(x)$	$\log(\frac{1}{2}-\phi_2(x))$	$\frac{3}{4}-\phi_4(x)$	$\log(\frac{3}{4}-\phi_4(x))$
0.8	5.94986 10^{-1}	2.57899 10^{-1}	$\bar{1}.41145$ -10376	3.66944 10^{-1}	$\bar{1}.56460$ - 4948	7.02732 10^{-1}	$\bar{1}.84679$ - 1806
0.9	5.01969	2.03092	.30769 -11096	3.27432	.51512 - 5845	6.74115	.82873 - 2469
1.0	4.15108	1.57299	.19673 -11931	2.86203	.45667 - 6757	6.36759	.80404 - 3213
1.1	3.36480	1.19795 10^{-1}	$\bar{1}.07742$ -12470	2.44961	.38910 - 7682	5.91369	.77186 - 4038
1.2	2.67344	8.96860 10^{-2}	$\bar{2}.95272$ -13323	2.05250	.31228 - 8612	5.38861	.73148 - 4915
1.3	2.08208	6.59920	.81949 -14084	1.68331	.22616 - 9546	4.81213	.68233 - 5832
1.4	1.58942	4.77149	.67865 -14851	1.35117	.13070 -10480	4.20743	.62401 - 6782
1.5	1.18930 10^{-1}	3.38948	.53014 -15627	1.06145 10^{-1}	$\bar{1}.02590$ -11416	3.58813	.55619 - 7753
1.6	8.72291 10^{-2}	2.36516	.37387 -16409	8.16091 10^{-2}	$\bar{2}.91174$ -12351	3.01059	.47866 - 8744
1.7	6.27110	1.62096	.20978 -17200	6.14092	.78823 -13283	2.46164	.39122 - 9741
1.8	4.41917	1.09095 10^{-2}	$\bar{2}.03778$ -17987	4.52273	.65540 -14214	1.96704	.29381 -10745
1.9	3.05247	7.2096 10^{-3}	$\bar{3}.85791$ -18788	3.26033	.51326 -15142	1.53590	-.18636 -11752
2.0	2.06670	4.6777	.67003 -19589	2.3006	.36184 -16068	1.1718 10^{-1}	$\bar{1}.06884$ -12759
2.1	1.37157 10^{-2}	2.9795	.47414 -20397	1.5891	.20116 -16992	8.7347 10^{-2}	$\bar{2}.94125$ -13765
2.2	8.92216 10^{-3}	1.8628	.27017 -21205	1.0746 10^{-2}	$\bar{2}.03124$ -17913	6.3620	.80360 -14769
2.3	5.68902	1.1432 10^{-3}	$\bar{3}.05812$ -22014	7.1140 10^{-3}	$\bar{3}.85211$ -18831	4.5280	.65591 -15769
2.4	3.55565	6.8862 10^{-4}	$\bar{4}.83798$ -22834	4.6110	.66380 -19747	3.1493	.49822 -16766
2.5	2.17828	4.0704	.60964 -23652	2.9264	.46633 -20663	2.1407	.33056 -17757
2.6	1.30805 10^{-3}	2.3611	.37312 -24473	1.8185	.25970 -21571	1.4223 10^{-2}	$\bar{2}.15299$ -18745
2.7	7.69925 10^{-4}	1.3440 10^{-4}	$\bar{4}.12839$ -25304	1.1066 10^{-3}	$\bar{3}.04399$ -22481	9.2371 10^{-3}	$\bar{3}.96554$ -19729
2.8	4.44208	7.5061 10^{-5}	$\bar{5}.87535$ -26133	6.5945 10^{-4}	$\bar{4}.81918$ -23392	5.8648	.76825 -20707
2.9	2.51211	4.1117	.61402 -26967	3.8482	.58526 -24298	3.6406	.56118 -21683
3.0	1.39253 10^{-4}	2.2098	.34435 -27795	2.1993	.34228 -25199	2.2098	.34435 -22652
3.1	7.56632 10^{-5}	1.1652 10^{-5}	$\bar{5}.06640$ -28631	1.2314 10^{-4}	$\bar{4}.09029$ -26105	1.3117 10^{-3}	$\bar{3}.11783$ -23617
3.2	4.02976	6.0268 10^{-6}	$\bar{6}.78009$ -29464	6.7490 10^{-5}	$\bar{5}.82924$ -27004	7.6148 10^{-4}	$\bar{4}.88166$ -24581
3.3	2.10372	3.0581	.48545 -30301	3.6241	.55920 -27903	4.3237	.63585 -25538
3.4	1.07649 10^{-5}	1.5222 10^{-6}	$\bar{6}.18244$ -31140	1.9062 10^{-5}	$\bar{5}.28017$ -28804	2.4015	.38047 -26492
3.5	5.39944 10^{-6}	7.4390 10^{-7}	$\bar{7}.87104$	9.8205 10^{-6}	$\bar{6}.99213$	1.3048 10^{-4}	$\bar{4}.11555$

* Note: $\log \phi_0'(x) = 0.0524550 - 0.4342945 x^2$

TABLE 2. (Continued)

x	$\phi_0'(x)$	$1 - \phi_0(x)$	$\log(1 - \phi_0(x))$	$\frac{1}{2} - \phi_2(x)$	$\log\left(\frac{1}{2} - \phi_2(x)\right)$	$\frac{3}{4} - \phi_4(x)$	$\log\left(\frac{3}{4} - \phi_4(x)\right)$
3.5	5.39944 10^{-6}	7.4309 10^{-7}	$\overline{7}.87104$ -31976	9.8205 10^{-6}	$\overline{6}.99213$ -29698	1.3048 10^{-4}	$\overline{4}.11555$ -27444
3.6	2.65459	3.5586	.55128 -32817	4.9562	.69515 -30593	6.9361 10^{-5}	$\overline{5}.84111$ -28390
3.7	1.27928 10^{-6}	1.6715 10^{-7}	$\overline{7}.22311$ -33660	2.4503	.38922 -31488	3.6075	.55721 -29335
3.8	6.04285 10^{-7}	7.7003 10^{-8}	$\overline{8}.88651$ -34504	1.1867 10^{-6}	$\overline{6}.07434$ -32383	1.8359 10^{-5}	$\overline{5}.26386$ -30277
3.9	2.79793	3.4791	.54147 -35349	5.6300 10^{-7}	$\overline{7}.75051$ -33274	9.1430 10^{-6}	$\overline{6}.96109$ -31215
4.0	1.26982 10^{-7}	1.5416 10^{-8}	$\overline{8}.18798$ -36194	2.6168	.41777 -34167	4.4560	.64894 -32151
4.1	5.64882 10^{-8}	6.7000 10^{-9}	$\overline{9}.82604$ -37038	1.1915 10^{-7}	$\overline{7}.07610$ -35056	2.1253 10^{-6}	$\overline{6}.32743$ -33083
4.2	2.46320	2.8555	.45566 -37884	5.3155 10^{-8}	$\overline{8}.72554$ -35945	9.9220 10^{-7}	$\overline{7}.99660$ -34014
4.3	1.05281 10^{-8}	1.1935 10^{-9}	$\overline{9}.07682$ -38735	2.3232 10^{-8}	$\overline{8}.36609$ -36834	4.5338	.65646 -34942
4.4	4.41077 10^{-9}	4.8918 10^{-10}	$\overline{10}.68947$ -39584	9.9483 10^{-9}	$\overline{9}.99775$ -37722	2.0279 10^{-7}	$\overline{7}.30704$ -35868
4.5	1.81131 10^{-9}	1.9662 10^{-10}	$\overline{10}.29363$ -40435	4.1738	.62053 -38610	8.8789 10^{-8}	$\overline{8}.94836$ -36792
4.6	7.29094 10^{-10}	7.7497 10^{-11}	$\overline{11}.88928$ -41284	1.7157 10^{-9}	$\overline{9}.23443$ -39497	3.8057	.58044 -37714
4.7	2.87667	2.9953	.47644 -42137	6.9099 10^{-10}	$\overline{10}.83947$ -40381	1.5970 10^{-8}	$\overline{8}.20330$ -38634
4.8	1.11253 10^{-10}	1.1352 10^{-11}	$\overline{11}.05507$ -42986	2.7268	.43566 -41269	6.5608 10^{-9}	$\overline{9}.81696$ -39552
4.9	4.21740 10^{-11}	4.2190 10^{-12}	$\overline{12}.62521$ -43839	1.0544 10^{-10}	$\overline{10}.02297$ -42150	2.6390	.42144 -40468
5.0	1.56708 10^{-11}	1.5375 10^{-12}	$\overline{12}.18682$ -44695	3.9946 10^{-11}	$\overline{11}.60147$ -43035	1.0394 10^{-9}	$\overline{9}.01676$ -41383
5.1	5.70762 10^{-12}	5.4938 10^{-13}	$\overline{13}.73987$ -45547	1.4829 10^{-11}	$\overline{11}.17112$ -43920	4.0081 10^{-10}	$\overline{10}.60293$ -42296
5.2	2.03766 10^{-12}	1.9249 10^{-13}	$\overline{13}.28441$ -46398	5.3942 10^{-12}	$\overline{12}.73192$ -44802	1.5135 10^{-10}	$\overline{10}.17997$ -43207
5.3	7.13056 10^{-13}	6.6135 10^{-14}	$\overline{14}.82043$ -47257	1.9227 10^{-12}	$\overline{12}.28390$ -45684	5.5963 10^{-11}	$\overline{11}.74790$ -44118
5.4	2.44584 10^{-13}	2.2277 10^{-14}	$\overline{14}.34786$ -48111	6.7151 10^{-13}	$\overline{13}.82706$ -46567	2.0264 10^{-11}	$\overline{11}.30672$ -45026
5.5	8.22332 10^{-14}	7.3579 10^{-15}	$\overline{15}.86675$ -48966	2.2982 10^{-13}	$\overline{13}.36139$ -47449	7.1855 10^{-12}	$\overline{12}.85646$ -45934
5.6	2.71007 10^{-14}	2.3828 10^{-15}	$\overline{15}.37709$ -49821	7.7073 10^{-14}	$\overline{14}.88690$ -48329	2.4953 10^{-12}	$\overline{12}.39712$ -46841
5.7	8.75442 10^{-15}	7.5662 10^{-16}	$\overline{16}.87888$ -50678	2.5328 10^{-14}	$\overline{14}.40361$ -49211	8.4862 10^{-13}	$\overline{13}.92871$ -47745
5.8	2.77197 10^{-15}	2.3556 10^{-16}	$\overline{16}.37210$ -51535	8.1565 10^{-15}	$\overline{15}.91150$ -50090	2.8266 10^{-13}	$\overline{13}.45126$ -48649
5.9	8.60328 10^{-16}	7.1904 10^{-17}	$\overline{17}.85675$ -52391	2.5739 10^{-15}	$\overline{15}.41060$ -50971	9.2207 10^{-14}	$\overline{14}.96477$ -49553
6.0	2.61730 10^{-16}	2.1520 10^{-17}	$\overline{17}.33284$	7.9595 10^{-16}	$\overline{16}.90089$	2.9461 10^{-14}	$\overline{14}.46924$

TABLES OF ERROR FUNCTION, ETC.

TABLE 2. (Continued)

x	$\phi_0'(x)$	$1 - \phi_0(x)$	$\log(1 - \phi_0(x))$	$\frac{1}{2} - \phi_2(x)$	$\log\left(\frac{1}{2} - \phi_2(x)\right)$	$\frac{3}{4} - \phi_4(x)$	$\log\left(\frac{3}{4} - \phi_4(x)\right)$
6.0	2.61730 10 ⁻¹⁶	2.1520 10 ⁻¹⁷	$\overline{17.33284}$ -53249	7.9595 10 ⁻¹⁶	$\overline{16.90089}$ -51851	2.9461 10 ⁻¹⁴	$\overline{14.46924}$ -50454
6.1	7.80473 10 ⁻¹⁷	6.3146 10 ⁻¹⁸	$\overline{18.80035}$ -54107	2.4120 10 ⁻¹⁶	$\overline{16.38238}$ -52730	9.2194 10 ⁻¹⁵	$\overline{15.96470}$ -51354
6.2	2.28127 10 ⁻¹⁷	1.8167 10 ⁻¹⁸	$\overline{18.25928}$ -54966	7.1628 10 ⁻¹⁷	$\overline{17.85508}$ -53609	2.8259 10 ⁻¹⁵	$\overline{15.45116}$ -52255
6.3	6.53593 10 ⁻¹⁸	5.1242 10 ⁻¹⁹	$\overline{19.70963}$ -55823	2.0844 10 ⁻¹⁷	$\overline{17.31899}$ -54488	8.4841 10 ⁻¹⁶	$\overline{16.92861}$ -53154
6.4	1.83549 10 ⁻¹⁸	1.4171 10 ⁻¹⁹	$\overline{19.15140}$ -56682	5.9444 10 ⁻¹⁸	$\overline{18.77411}$ -55366	2.4950 10 ⁻¹⁶	$\overline{16.39707}$ -54052
6.5	5.05258 10 ⁻¹⁹	3.8422 10 ⁻²⁰	$\overline{20.58458}$ -57543	1.6613 10 ⁻¹⁸	$\overline{18.22045}$ -56244	7.1870 10 ⁻¹⁷	$\overline{17.85655}$ -54949
6.6	1.36330 10 ⁻¹⁹	1.0213 10 ⁻²⁰	$\overline{20.00915}$ -58399	4.5500 10 ⁻¹⁹	$\overline{19.65801}$ -57123	2.0280 10 ⁻¹⁷	$\overline{17.30706}$ -55846
6.7	3.60559 10 ⁻²⁰	2.6617 10 ⁻²¹	$\overline{21.42516}$ -59259	1.2212 10 ⁻¹⁹	$\overline{19.08678}$ -58000	5.6053 10 ⁻¹⁸	$\overline{18.74860}$ -56741
6.8	9.34713 10 ⁻²¹	6.8009 10 ⁻²²	$\overline{22.83257}$ -60120	3.2120 10 ⁻²⁰	$\overline{20.50678}$ -58877	1.5177 10 ⁻¹⁸	$\overline{18.18119}$ -57637
6.9	2.37517 10 ⁻²¹	1.7036 10 ⁻²²	$\overline{22.23137}$ -60980	8.2795 10 ⁻²¹	$\overline{21.91801}$ -59755	4.0255 10 ⁻¹⁹	$\overline{19.60482}$ -58530
7.0	5.91597 10 ⁻²²	4.1838 10 ⁻²³	$\overline{23.62157}$ -61840	2.0915 10 ⁻²¹	$\overline{21.32046}$ -60632	1.0460 10 ⁻¹⁹	$\overline{19.01952}$ -59424
7.1	1.44435 10 ⁻²²	1.0073 10 ⁻²³	$\overline{23.00317}$ -62700	5.1777 10 ⁻²²	$\overline{22.71414}$ -61509	2.6624 10 ⁻²⁰	$\overline{20.42528}$ -60318
7.2	3.45644 10 ⁻²³	2.3778 10 ⁻²⁴	$\overline{24.37617}$ -63561	1.2562 10 ⁻²²	$\overline{22.09905}$ -62385	6.6390 10 ⁻²¹	$\overline{21.82210}$ -61209
7.3	8.10779 10 ⁻²⁴	5.5025 10 ⁻²⁵	$\overline{25.74056}$ -64422	2.9868 10 ⁻²³	$\overline{23.47520}$ -63261	1.6218 10 ⁻²¹	$\overline{21.21001}$ -62102
7.4	1.86418 10 ⁻²⁴	1.2484 10 ⁻²⁵	$\overline{25.09634}$ -65282	6.9597 10 ⁻²⁴	$\overline{24.84259}$ -64136	3.8814 10 ⁻²²	$\overline{22.58899}$ -62992
7.5	4.20136 10 ⁻²⁵	2.7767 10 ⁻²⁶	$\overline{26.44352}$ -66144	1.5894 10 ⁻²⁴	$\overline{24.20123}$ -65013	9.1006 10 ⁻²³	$\overline{23.95907}$ -63883
7.6	9.28123 10 ⁻²⁶	6.0545 10 ⁻²⁷	$\overline{27.78208}$ -67005	3.5572 10 ⁻²⁵	$\overline{25.55110}$ -65888	2.0905 10 ⁻²³	$\overline{23.32024}$ -64773
7.7	2.00972 10 ⁻²⁶	1.2943 10 ⁻²⁷	$\overline{27.11203}$ -67867	7.8023 10 ⁻²⁶	$\overline{26.89222}$ -66764	4.7045 10 ⁻²⁴	$\overline{24.67251}$ -65661
7.8	4.26559 10 ⁻²⁷	2.7124 10 ⁻²⁸	$\overline{28.43336}$ -68729	1.6772 10 ⁻²⁶	$\overline{26.22458}$ -67640	1.0373 10 ⁻²⁴	$\overline{24.01590}$ -66552
7.9	8.87435 10 ⁻²⁸	5.5728 10 ⁻²⁹	$\overline{29.74607}$ -69590	3.5333 10 ⁻²⁷	$\overline{27.54818}$ -68516	2.2407 10 ⁻²⁵	$\overline{25.35038}$ -67439
8.0	1.80971 10 ⁻²⁸	1.1224 10 ⁻²⁹	$\overline{29.05017}$	7.2950 10 ⁻²⁸	$\overline{28.86302}$	4.7423 10 ⁻²⁶	$\overline{26.67599}$