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COMPARISON AND COMMENT OF TWO SIMULTANEOUS OBSERVATION METHODS FOR SATELLITE TRIANGULATION

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Abstract

In order to determine geodetic position by use of passive satellite, the trailing method and the simultaneous method are generally employed. The trailing method of Hirose does not necessitate timing device except at one of base stations, while the simultaneous method requires precise timing device at each station.

Investigation on reduction techniques of both methods shows that the trailing method has the following features in comparison with the simultaneous method:

(1) Timing accuracy required at the base station is only 1/15 sec and hence the delay of propagation of time signal can generally be disregarded in reduction process.

(2) Planetary aberration effect is not so serious.

(3) Parallactic refraction effect can briefly be treated by introducing the refraction height.

(4) Resulting accuracy depends on the geometrical relations of the base stations to the satellite track, accordingly weight should be assigned to each observation equation obtained from simultaneous observation at three stations.

Moreover, a new smoothing method for dancing images of satellite is proposed and compared to Hirose's smoothing method by preliminary orbit.

Experimental investigation on observation techniques shows that systematic error may exist in the measured positions of satellite trail, being caused by magnitude differences between comparison stars and satellite, in case their differences are large.

In conclusion, due to its advantages in handy equipments and in economy, the trailing method seems to be preferable for determining geodetic positions of off-shore islands and secluded inland spots of poor facility of transportation.

1. Introduction

In order to make the satellite triangulation by observing the negative satellite with the ordinary simultaneous method, the camera at each station is required to be equipped with timing device of high precision. Assuming that a satellite is moving at an altitude of 1000 km, an error of 1″ in arc in deriving the satellite's position produces an error of 5 meters for determining geodetic positions. The apparent angular speed of this satellite when passing near the zenith of observing stations is about 1500″ per sec. In order to determine the geodetic positions within the error of ±5 m, accordingly, an accuracy of 0.6 msec in timing is needed. To meet this requirement, several kinds of timing devices have been developed by many investigators.

Hirose (1962, 1963) proposed a kind of simultaneous method, called as "Trailing Method", which does not necessitate the precise timing device at each station but only one of the base stations of known coordinates is required to have a timing device. The author and his collaborators have developed practical techniques of observation and reduction for this method, and have made several experiments in laboratory and in field.

In the present paper, several problems on the reduction techniques and on the significant error sources are discussed. Finally, a summary of the relative advantages of the trailng method and the ordinary simultaneous method is provided.

2. Vector expressions of the principle

For convenience of discussions the principles of the ordinary simultaneous method and of the trailing method are given in vector expressions below apart from practical reduction methods.

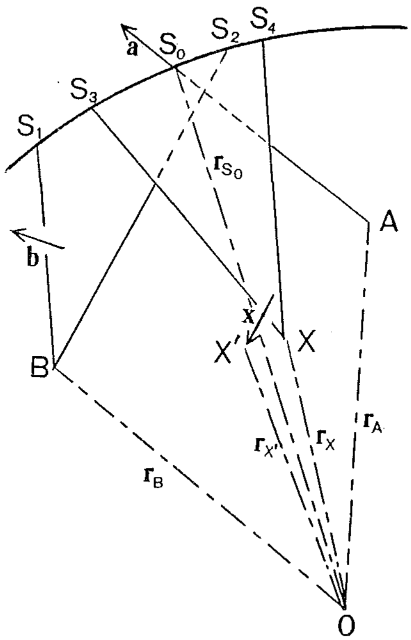

Fig. 1. Principle of trailing method.

(1) Trailing method

Notations and symbols: (Fig. 1)

A : one base station having a camera with a timing device,

B : another base station having simple camera on a equatorial mounting without any timing device,

X: an unknown station having the same camera as the station B whose geodetic position is aimed to be determined,

S_0: the satellite position in space at time t,

X' : the assumed position of the station X,

O : the centre of the reference earth spheroid,

S_1S_2 and S_3S_4: sedgements of the satellite trail in space containing S_0 as seen from B and X, respectively,

r_A, r_B, r_X and $r_{X'}$: radius vectors of the stations A, B, X and X' respectively,

r_{S_0}: radius vector of the satellite S_0.

At three stations A, B and X observations are made simultaneously by pointing the cameras respectively to the predicted directions of the satellite for a given time in such a way that the satellite passes the centre of the camera field during the exposure interval at each station. Hence, the images of the trail segments S_1S_2, and S_3S_4 photographed respectively at B and X contain the position S_0 at t, if the predictions are enough accurate.

Since the camera at A has a timing device, the direction of S_0 referred to celestial system is derived from a series of chopped images of the satellite around t by interpolation. We shall denote the unit vector of this direction by $\boldsymbol{a}$.

Next, we must determine the planes BS_1S_2 and XS_3S_4. If the distances S_0S_1, S_0S_2, S_0S_3 and S_0S_4 are required to be short so that these arc segments can be approximated by straight lines respectively. Though we have not any information on the position of S_0 on these segements, it can be roughly obtained by an approximation as described below. Then, we can take the positions S_1, S_2, S_3 and S_4 adjacent to this approximated position of S_0. Whence, the directions $\overrightarrow{BS_1}$, $\overrightarrow{BS_2}$; $\overrightarrow{XS_3}$, $\overrightarrow{XS_4}$ are obtained in the celestial coordinates by measuring the images of S_1, S_2, S_3 and S_4. It must be noted here that these directions do not show those which will be seen from the coordinate system fixed to the earth due to the diurnal motion of the celestial sphere. In practice, however, this will not be so serious as far as we concern to the determination of the satellite position S_0 at the time t, as stated later.

Next, we calculate normal directions $\boldsymbol{b}$ and $\boldsymbol{x}$ of the planes BS_1S_2 and XS_3S_4 by

$$\boldsymbol{b} = \overrightarrow{BS_1} \times \overrightarrow{BS_2} \tag{1}$$

$$\boldsymbol{x} = \overrightarrow{XS_3} \times \overrightarrow{XS_4}. \tag{2}$$

Basing on these preparations, we will derive the observation equation for determing the position of X as in the following.

We have for radius vector $\boldsymbol{r}$ of any point on the plane BS_1S_2

$$\boldsymbol{b}(\boldsymbol{r} - \boldsymbol{r}_B) = 0, \tag{3}$$

and for any point on the straight lins AS_0

$$\boldsymbol{r} = \boldsymbol{r}_A + a\boldsymbol{a}, \tag{4}$$

where a is a parameter.

Since $\boldsymbol{r}_{S_0}$ is a radius vector denoting the intersection point S_0 of the plane BS_1S_2 with the straight line AS_0, eliminating $\boldsymbol{r}$ from (3) and (4), we get

$$\boldsymbol{r}_{S_0} = \boldsymbol{r}_A + \boldsymbol{a}(\boldsymbol{b} \cdot \boldsymbol{r}_{BA})/(\boldsymbol{b} \cdot \boldsymbol{a}), \tag{5}$$

where $\boldsymbol{r}_{BA} = \boldsymbol{r}_B - \boldsymbol{r}_A.$

On the other hand, the equation of the plane XS_3S_4 is given by

$$\boldsymbol{x}(\boldsymbol{r}-\boldsymbol{r}_{S_0})=0. \tag{6}$$

Since X lies in this plane, we obtain the observation equation

$$\boldsymbol{x}(\boldsymbol{r}_X-\boldsymbol{r}_{S_0})=0. \tag{7}$$

Since (7) contains three unknown components of vector $\boldsymbol{r}_X$, three or more sets of simultaneous observations are necessary to solve it. We shall call the plane defined by (7) as "plane of position" or "position plane" hereafter.

In practical techniques of reduction, however, it seems to be more convenient to employ the following method which defines the position plane by an assumed position X'.

The equation of the straight line which is perpendicular to the plane XS_3S_4 and passes through the assumed position X' is given by

$$\boldsymbol{r}=\boldsymbol{r}_{X'}+x\boldsymbol{x}, \tag{8}$$

where x is a parameter.

Let p be the distance of X' from the plane XS_3S_4. Eliminating $\boldsymbol{r}$ from (6) and (8), p is given by the following equation,

$$p=(\boldsymbol{x}\cdot\boldsymbol{r}_{S_0X'}), \tag{9}$$

where $$\boldsymbol{r}_{S_0X'}=\boldsymbol{r}_{S_0}-\boldsymbol{r}_{X'}.$$

As already stated, $\boldsymbol{x}$ is determined from the reduction of the photographic plate taken at station by (2), $\boldsymbol{r}_{X'}$ is the assumed radius vector, and $\boldsymbol{r}_{S_0}$ can be computed by (5). Then the position plane is defined as a plane which lies at the distance p from the assumed postion X' and have the normal direction $\boldsymbol{x}$.

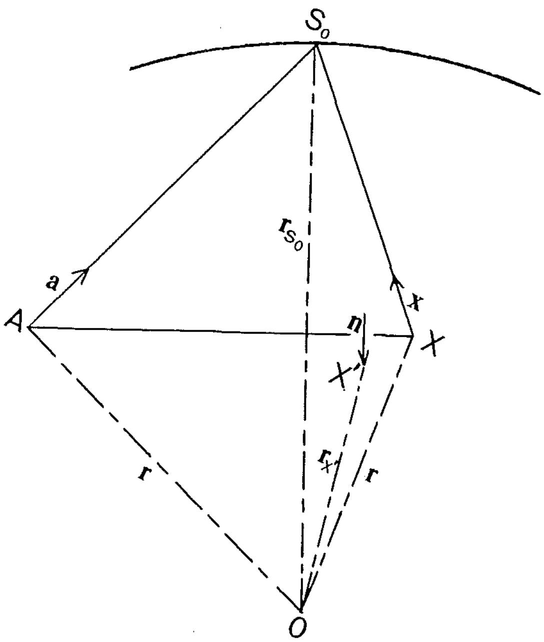

Fig. 2. Principle of Simultaneous method.

(2) Simultaneous method

Notations and symbols: (Fig. 2)

A : a base station having a camera equipped with a precise timing device,
X : an unknown station having the same camera as A,
X': the assumed position of station X,
S_0 : the satellite position in space at time t,
O : the centre of the reference earth spheroid,
r_A, r_X and $r_{X'}$: radius vectors of stations A, X and X' respectively,
r_{S_0}: radius vector of the satellite S_0.

In this method, observations at the two stations need not to be strictly simultaneous with each other, provided that the duration of a series of exposures made at one of the stations contains a part of those made at another station. We shall denote the unit vectors of the satellite directions as seen from stations A and X at the same time t by $\boldsymbol{a}$ and $\boldsymbol{x}$, and they can be derived from these simultaneous data with timing by interpolation. If the observations are free from errors, then the straight line AS_0 passing through A in the direction $\boldsymbol{a}$ and the straight line XS_0 passing through X in the direction $\boldsymbol{x}$ should meet at the point S_0 in space. Hence, the plane which contains a group of straight lines intersecting the straight line AS_0 respectively with the direction vector $\boldsymbol{x}$ must contain the unknown point X. Then, this plane is a "position plane" analogous to the case of trailing method.

Hence, the two position planes obtained by two sets of simultaneous observations for respective two positions of satellite can fix the straight line AX at the intersection of these planes.

Consider now another known station B. By a similar simultaneous observation at B and X we can get another position plane $BS_0'X$, where S_0' denotes the satellite position in space at an another time t'.

Whence the position of X is fixed as the intersection of the position plane $BS_0'X$ with the straight line AX.

So far we have implicitly assumed that stations A and B belong to the same geodetic system. If only the length of the line AB is previously given, however, this assumption is not necessarily, because the coordinates of B relative to A can be derived from two or more sets of simultaneous observations at A and B using the known length of AB.

The equation of the plane AS_0X is given by

$$\boldsymbol{n}(\boldsymbol{r}-\boldsymbol{r}_A)=0, \tag{10}$$

where $\boldsymbol{n}$ is a normal vector of the plane.

From definition, $\boldsymbol{n}$ is expressed by the equation

$$n \cdot \boldsymbol{n}=\boldsymbol{a}\times\boldsymbol{x}, \tag{11}$$

where n is sine of the angle subtended between two straight lines AS_0 and XS_0, and the following relation is held between n and the components of the vectors $\boldsymbol{a}$ and $\boldsymbol{x}$,

$$n^2=\sin^2\theta=\begin{vmatrix} a_y & x_y \\ a_z & x_z \end{vmatrix}^2+\begin{vmatrix} a_z & x_z \\ a_x & x_x \end{vmatrix}^2+\begin{vmatrix} a_x & x_x \\ a_y & x_y \end{vmatrix}^2. \tag{12}$$

The straight line perpendicular to the plane AS_0X and passing through the assumed position X', is given by the following equation:

$$\boldsymbol{r} = \boldsymbol{r}_{X'} + m\boldsymbol{n}, \tag{13}$$

m being a parameter.

Let p be the vertical distance of X' from the plane AS_0X. Eliminating $\boldsymbol{r}$ from (10) (13), we can get p

$$p = \boldsymbol{n}(\boldsymbol{r}_A - \boldsymbol{r}_{X'}). \tag{14}$$

Since the right hand side of (14) are already known, we can calculate p from (14). Then the position plane AS_0X is established as the plane which lies at the vertical distance p from the assumed position X' and have the normal direction $\boldsymbol{n}$.

3. Optimum conditions for observations and weights of the position plane

In this section, we shall consider about the optimum conditions for geometrical relations of observing stations relative to a satellite so as to minimize resulting errors, and shall introduce weights to be assigned to the position planes.

(1) Trailing method

As is clearly seen from Fig. 1, in this case the resulting accuracy is affected by the angle which the line AS_0 makes with the plane BS_1S_2.

In order to see this effect quantitatively, we derive the following error equations by differentiating (5) and (9), respectively.

$$\delta\boldsymbol{r}_{S_0} = \delta\boldsymbol{r}_A + \delta\boldsymbol{a}(\boldsymbol{b}\cdot\boldsymbol{r}_{BA})/(\boldsymbol{b}\cdot\boldsymbol{a}) - (\boldsymbol{b}\cdot\delta\boldsymbol{a})(\boldsymbol{b}\cdot\boldsymbol{r}_{BA})\boldsymbol{a}/(\boldsymbol{b}\cdot\boldsymbol{a})^2$$
$$+ \{(\delta\boldsymbol{b}\cdot\boldsymbol{r}_{BA})(\boldsymbol{b}\cdot\boldsymbol{a}) - (\boldsymbol{b}\cdot\boldsymbol{r}_{BA})(\delta\boldsymbol{b}\cdot\boldsymbol{a})\}\boldsymbol{a}/(\boldsymbol{b}\cdot\boldsymbol{a})^2 + (\boldsymbol{b}\cdot\delta\boldsymbol{r}_{BA})\boldsymbol{a}/(\boldsymbol{b}\cdot\boldsymbol{a}), \tag{15}$$

and

$$\delta p = (\delta\boldsymbol{x}\cdot\boldsymbol{r}_{S_0X'}) + \boldsymbol{x}(\delta\boldsymbol{r}_{S_0} - \delta\boldsymbol{r}_{X'}). \tag{16}$$

Substituting in $\delta\boldsymbol{r}_{S_0}$ of (16) the right hand side of (15), we obtain

$$\delta p = (\boldsymbol{x}\cdot\delta\boldsymbol{r}_{AX'}) + (\boldsymbol{b}\cdot\delta\boldsymbol{r}_{BA})(\boldsymbol{a}\cdot\boldsymbol{x})/(\boldsymbol{b}\cdot\boldsymbol{a}) + \delta\boldsymbol{x}\cdot\boldsymbol{r}_{S_0X'} + (\boldsymbol{x}\cdot\delta\boldsymbol{a})(\boldsymbol{b}\cdot\boldsymbol{r}_{BA})/(\boldsymbol{b}\cdot\boldsymbol{a})$$
$$+ (\boldsymbol{a}\cdot\boldsymbol{x})(\delta\boldsymbol{b}\cdot\boldsymbol{r}_{BA})/(\boldsymbol{b}\cdot\boldsymbol{a}) - (\boldsymbol{a}\cdot\boldsymbol{x})(\boldsymbol{b}\cdot\boldsymbol{r}_{BA})\{(\delta\boldsymbol{b}\cdot\boldsymbol{a}) + (\boldsymbol{b}\cdot\delta\boldsymbol{a})\}/(\boldsymbol{b}\cdot\boldsymbol{a})^2, \tag{17}$$

In this equation, we can regard differentials $\delta\boldsymbol{a}$, $\delta\boldsymbol{b}$, $\delta\boldsymbol{x}$, $\delta\boldsymbol{r}_{AX'}$, $\delta\boldsymbol{r}_{BA}$ and δp as the following quantities:

$\delta\boldsymbol{a}$ = positioning error of satellite at A,

$\delta\boldsymbol{b}$ = positioning error of trailed satellite path at B,

$\delta\boldsymbol{x}$ = positioning error of trailed satellite path at X,

$\delta\boldsymbol{r}_{BA}$, $\delta\boldsymbol{r}_{AX'}$ = relative position errors between A and B, and X' and A, due to the adopted geodetic coordinates erros, including the errors caused by incorrectness of the reference earth spheroid, and

δp = the resulting error of the position plane.

The sixth term of the right hand side of (17) has the square of $(\boldsymbol{b}\cdot\boldsymbol{a})$ in the denominator. Then, δp increases rapidly as $(\boldsymbol{b}\cdot\boldsymbol{a})$ becomes small, that is, as the direction joining A and B approaches to the sub-track of the satellite on the earth's surface. This effect is significant when the distance between A and B

much smaller than the range of the satellite.

From this reason, it is highly desirable to make the observations under the condition that the sub-track of satellite crosses perpendicularly to the straight line AB at its centre. However, as courses of satellites are fixed by their launching conditions, we must select the proper satellites having such inclinations that satisfies the above condition for the locations of A and B.

In practice, there are little chances that the above condition is completely satisfied. Hence, in order to derive the final position of the unknown station from these position planes, weights should be assigned to these position planes according as the degree of satisfying the condition.

As those weights, it seems to be reasonable to take the following one based on the error equation (17).

Separating terms of observation errors in equation (17) for respective stations, we define the observation error proper to each station as follows:

$$
\begin{aligned}
E_A &= \sqrt{\{(\boldsymbol{b}\cdot\boldsymbol{r}_{BA})/(\boldsymbol{b}\cdot\boldsymbol{a})\}^2+\{(\boldsymbol{a}\cdot\boldsymbol{x})(\boldsymbol{b}\cdot\boldsymbol{r}_{BA})/(\boldsymbol{b}\cdot\boldsymbol{a})^2\}^2}\times \varDelta A,\\
E_B &= \sqrt{\{(\boldsymbol{a}\cdot\boldsymbol{x})\cdot|\boldsymbol{r}_{BA}|/(\boldsymbol{b}\cdot\boldsymbol{a})\}^2+\{(\boldsymbol{a}\cdot\boldsymbol{x})(\boldsymbol{b}\cdot\boldsymbol{r}_{BA})/(\boldsymbol{b}\cdot\boldsymbol{a})^2\}^2}\times \varDelta B, \qquad (18)\\
E_X &= |\boldsymbol{r}_{X'S}|\times \varDelta X,
\end{aligned}
$$

where $\varDelta A$, $\varDelta B$ and $\varDelta X$ denote the positioning errors of satellite observed at A, B and X, respectively.

Assuming that the geodetic coordinates of the base stations are free from errors, the resulting error of the position plane can be given by the following equation,

$$E = \sqrt{E^2_A+E^2_B+E^2_X.} \qquad (19)$$

We can take respective reciprocals of the weights to be assigned to the position planes.

(2) Simultaneous method

Analogous to the trailing method, we can derive the error equations of the position plane in this case from (11) and (14).

Differentiating (11) and (14), we obtain

$$
\begin{aligned}
\delta p &= (\delta\boldsymbol{n}\cdot\boldsymbol{r}_{AX'})+(\boldsymbol{n}\cdot\delta\boldsymbol{r}_{AX'}),\\
\delta\boldsymbol{n}\cdot\boldsymbol{n}+\boldsymbol{n}\cdot\delta\boldsymbol{n} &= \delta\boldsymbol{a}\times\boldsymbol{x}+\boldsymbol{a}\times\delta\boldsymbol{x}. \qquad (20)
\end{aligned}
$$

Here, we will regard $\delta\boldsymbol{a}$, $\delta\boldsymbol{x}$, $\delta\boldsymbol{n}$, $\delta\boldsymbol{r}_{AX'}$ and δp as the following quantities:

$\delta\boldsymbol{a}$ = positioning error of a satellite at A,

$\delta\boldsymbol{x}$ = positioning error of a satellite at X,

$\delta\boldsymbol{n}$ = error derived from the relation between $\delta\boldsymbol{a}$ and $\delta\boldsymbol{x}$ by (11),

$\delta\boldsymbol{r}_{AX'}$ = relative position error between X' and A, due to the adopted geodetic coordinates errors including the errors caused by incorrectness of the reference earth spheroid,

δp = the resulting error of the position plane.

In this case, as we can easily see in Fig. 2, δp is independent of the course of satellite, but at first sight the error of the position plane appears to be affected by the distance both stations relative to the slant range of the satellite.

To examine this problem, we will investigate behaviour of δp for the case of $n \to 0$ which corresponds to $A \to C$.

Inserting $\delta \boldsymbol{n}$ of the lower equation of (20) to its upper equation, and assuming that $\delta r_{AX}'=0$, we get

$$\delta p = (\delta \boldsymbol{a} \times \boldsymbol{x} + \boldsymbol{a} \times \delta \boldsymbol{x} - \delta n \cdot \boldsymbol{n}) \boldsymbol{r}_{AX}'/n. \tag{21}$$

Then

$$\lim_{n \sim 0} |\boldsymbol{r}_{AX}'| \to |\boldsymbol{r}_{SX}'| \sin \theta. \tag{22}$$

Inserting (12) and (22) into (21) and considering that $\delta n = \cos\theta \ \delta\theta$, the following equation holds,

$$\delta p_{\lim \theta \sim 0} = |(\delta \boldsymbol{a} \times \boldsymbol{x} + \boldsymbol{a} \times \delta \boldsymbol{x} - \boldsymbol{n} \cdot \delta\theta)| \cdot |\boldsymbol{r}_{SX}'| \cdot \cos\varphi, \tag{23}$$

where φ is the angle which $\boldsymbol{r}_{AX}'$ makes with the vector expressed by the bracket of above equation.

As is clearly seen from (23), the accuracy of determining the position plane depends mainly on the range of satellite, and it does not depend on the distance between the both stations against the above expectation.

Of course, in order to determine the final positions of the unknown station with much more accuracy from these position planes, it is strongly desirable that these position planes meet at right angles each other.

Analogous to the case of the trailing method, we can derive the weight to be assigned to the position planes from the error equation (21), It is the reciprocal of the resulting error E of the position plane and

$$E = \sqrt{\varDelta A^2 + \varDelta X^2} \cdot |\boldsymbol{r}_{AX}'| / n, \tag{24}$$

where $\varDelta A$ and $\varDelta X$ denote the observation error at A and X.

In order to increase the probability of succesful observations, we can employ the trailing method with the simultaneous method for one simultaneous data obtained at unknown station. In this case the camera needs the precise timing device. For the reduced position planes, the weights defined by (19) and (24) should be assigned, respectively

4. Timing accuracy

We have already seen that the trailing method needs a timing device of medium precision at only one base station. In this section, we shall consider about the accuracy of the timing device.

See Fig. 1 again. As the figure shows the geometrical relation between observing stations and a satellite path in space as seen from a coordinate system fixed on the earth, the satellite position at the observed time $(t+\varDelta t)$ and that at the correct time t are both to be contained in the illustrated satellite path, where $\varDelta t$ means the timing error at the base station A.

As already stated, in the trailing method the unknown station is fixed by the position planes which contain these satellite paths. Accordingly, if the positioning error of the satellite path caused by the timing error $\varDelta t$, which is equivalent to the diurnal motion of the celestial sphere during Δt, does not exceed the measured error of the photographic plate, and even if the satellite position reduced from the photographic plate taken at A does not correspond to the observed time $(t+\Delta t)$, but to the time t, we can get the same position plane that would be derived from the true satellite position at $(t+\Delta t)$, within the limits of the error.

Since the diurnal motion of the celestial sphere reaches a maximum value, 15″/sec, on the celestial equator, it is enough to take the order of 1/15 sec as the timing accuracy necessary to the base station A, assuming an accuracy of 1″ in the measurement of the photographic plate.

On the other hand, in the simultaneous method, the timing device requires the timing accuracy of better than 1 msec at least, because the movement of satellite in the time intervals corresponding to the timing error give rise to the discrepancy between the directions of satellite as seen from each station, as is clearly seen from Fig. 2.

5. Effects of planetary aberration and diurnal aberration

Since the velocity of light is finite, the light emitted from the satellite at time t is received by the cameras at A, B and X (See Figs. 1 and 2) respectively Δt_A $(=AS_0/c)$, Δt_B $(=BS_0/c)$ and Δt_X $(=XS_0/c)$ later, where c is the velocity of light. Thus, owing to the orbital motion of the satellite and the displacement of the positions of the ground stations caused by the earth rotation during this traveling time of light, the directions of satellite as seen from the ground stations at the time of observations do not show the true directions for the satellite position in space at the time.

In astronomy, this effect is well known as aberration effect, i. e. planetary aberration and diurnal aberration. In the satellite triangulation, as is clearly seen from the principle of section 2, there is no necessity for knowing the satellite true position at the time of observation at the ground stations, so long as the directions of the satellite as seen from each station correspond to the same satellite position at the same time, and we have only to take account of the relative differences in the aberration effect for each station.

In general, the corrections of the aberration effect are given for the directions of the satellite. For the diurnal aberration, however, we can employ the method to correct the coordinates of the station by the quantity corresponding to the earth rotation during the traveling time of light. This method is especially convenient for the trailing method from the following reasons.

In the trailing method the timing accuracy is not too serious, and the effect of planetary aberration alone is equivalent to the effect caused by the timing error. So we meet not to take account of the planetary aberration, but merely the diurnal aberration effect for the coodinates of the station.

6. The effects of atmospheric refraction

In astronomical observations, as is well known, the light from heavenly body has to pass through the earth's atmosphere before reaching the observer at the earth's surface and during its passage a ray of light suffers a change in direction owing to refraction. The case is same in satellite observations, too.

For this reason, the actual images of satellite and background stars on the photographic plate are displaced from the position it would occupy geometrically.

As far as we aim to determine the celestial coordinates of satellite with reference to background stars, the treatment of the problem is very simple. We can generally employ the method of plate constant well known in astrometry without any consideration of the theoretical formulae on the effect of refraction.

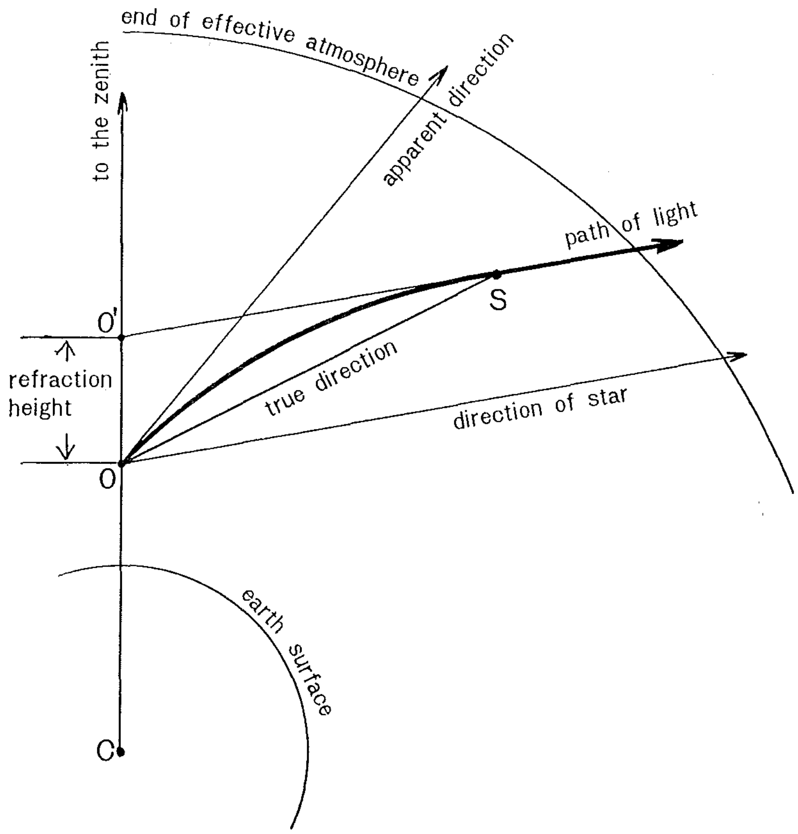

Fig. 3. Atmospherc refraction.

However, in satellite triangulation problem, satellite is treated as an object being situated at a finite distance from observers. Then, amount of refraction to the satellite differs from that to the background stars which lie practically in infinite distance.

In Fig. 3, let C be the centre of the earth, O the observer, and S the satellite. Owing to the atmospheric refraction, the light from the satellite reaches to the ground observer along the curve denoted as "path of light" in the figure.

Consequently, the satellite will be seen by the observer in the direction along the tangent to the path of light at O. We shall call this direction as "apparent direction". On the other hand the direction of satellite obtained directly from plate reduction shows such one that would be seen by the observer, if there were no atmospherical refraction and the satellite was regarded as at an infinite distance. In Fig. 3, this direction is denoted by "direction of star", and its direction is parallel to the part of straight line of the path of light in outer atmosphere. Accordingly, for the satellite of a finite distance, the direction derived from such plate reduction, the direction of star, differs from "true direction" joining the satellite to the observer by the depicted angle. Taking these refraction effects into account, there are two methods to solve the satellite triangulation problem.

One of them treats the problem with the true direction of the satellite as seen from the observers O which is derived by correcting "parallactic refraction angle Σ" to the apparent direction, and the other treats the problem with the direction of the satellite as seen from the intersection point O' of the tangent to the path of light at S with the vertical line at O.

The correction OO' for the height of the observer in the latter case is the same as one which is defined as "refraction height" in spherical astronomy, assuming that the heavenly body is situated outside the atmosphere.

In such case as the satellite is outside the atmosphere, —this condition is nearly satisfied as far as it has a height of over 100 km—, we can, therefore, apply the method of the refraction height directly to our satellite triangulation problem. The refraction heights depend on the altitude of heavenly body and the physical state of the atmosphere, and their numerical values have already been published by various authors, e. g. Chauvenet (1891) and Oterma (1960). The following provides one evaluated by Oterma.

TABLE 1. REFRACTION HEIGHTS

z_0'	h	z_0'	h	z_0'	h
0°	2.33 m	40°	3.96 m	80°	70.8 m
5	2.35	45	4.65	81	85.6
10	2.40	50	5.62	82	105.5
15	2.50	55	7.04	83	132.9
20	2.64	60	9.24	84	171.9
25	2.84	65	12.88	85	229.7
30	3.11	70	19.50		
35	3.47	75	33.44	90	2215

In this table, z_0' denotes the apparent zenith distance of the heavenly body, and h denotes the refraction height.

On the other hand, investigations on the parallactic refraction have been promoted by many authors since the launching of artificial satellites, and its several solutions have been published up to this time (Oterma, 1960; Schmid, 1959, 1962; Veis, 1960 etc.).

Of course, we can easily calculate the refraction height from the parallactic refraction for such special case as the satellite is outside the atmosphere. In this case, however, it should be noted that the refraction heights are indepedent of the distance of satellite, although the parallactic refraction is not so.

In the trailing method, a treatment with this refraction height is especially convenient, because it does not necessitate any corrections for the satellite positions observed as well as the case of the planetary aberration. However, in case we aim to determine a precise satellite orbit from numerous observation data, the treatment with the parallactic refraction is rather convenient, because, if it is not the case, we must employ the various heights corrected by the refraction height for the same station from observations to observations.

7. Seeing effect

In the satellite triangulation, in general, the satellite positions are determined with help of background stars. Hence, in satellite astrometry, it is to be especially noted that the duration of the exposure for star images and satellite image may differ from each other very much and may occur at widely different times.

As a result, the image motion of satellite caused by the atmospheric disturbances in the neighbourhood of camera may affect the mean position of the trailed image, while the star images are averaged about their true positions in case of using an equatorial mounting camera.

Occasionally, individual point on the satellite trailed image may appeares away from the true position in the order of several seconds of arc (Hyneck, 1960). From this reason, we must be careful to smooth the measured positions of satellite.

For serial satellite images photographed on one photographic plate, in general, the smoothing method by polynomial approximation is adopted. However, if we determine the satellite position at any time from a rather small number of films photographed over wide field, such as Baker Nunn Camera, this method seems to be unsuitable.

Recently, the author has proposed a smoothing method by a modified polynomial approximation applicable to such cases. The outline is given below.

First, we obtain the celestial coordinates of the satellite position on each film, and transform their coordinates to the standard coordinates ξ, ζ, of which the origin is taken to the point corresponding to a central satellite image. Next, the standard coordinates ξ, ζ are transformed to rectangular coordinates x, y, where x axis of the coordinates is taken nearly parallel to the direction of movement of the satellite.

Assuming that the relation between x and y coordinates of serial satellite images is given in a polynomial form, we derive the coefficients of the polynomial by the method of least squares, where numbers of terms of the polynomial should be determined by taking account of the standard deviation. In practice it will be sufficient to take a 2nd- to 3rd- order polynomial. As obviously seen from the definition of the x, y coordinates system, variation of y coordinates to time is very small. Accordingly, we have only to take account of variation of x coordinates with time. Assuming, again, a relation in a polynomial form between the x coordinates and time, we derive the coefficients of the polynomial by the method of least-squares. Finally, the smoothed satellite position at any time can be obtained by tracing the above twe polynomials in a reverse order.

A feature of this smoothing process is found in the fact that satellite positions and their exposured times are simultaneously smoothed, though the usual method smoothes x and y coordinates with respect to time independently with each other, and the author's present method is practical for smoothing positions of serial chopped images of a satellite photographed with timing device on the same plate, too.

Besides, Hirose (1962) has proposed the following another smoothing method. First, the preliminary orbital elements of the satellite are calculated from the celestial positions of three satellite images selected out of serial ones. From the orbital elements, the positions of serial satellite images are predicted for each exposured time, and plot the differences $(O-C)$ between the predicted positions and the observed positions on a graph. By smoothing these $(O-C)$ values a curve can be drawn, and we can read from this curve the corrections which should be applied to the observed positions to get the most probable satellite positions for a given time.

The following table provides some examples of comparison between the above two smoothing methods. The data used for the smoothing are of consecutive 6~8 frames of films taken by Baker Nunn Camera of Tokyo Astronomical Observatory for Echo 2.

TABLE 2. COMPARISON OF TWO SMOOTHING METHODS

No.	Satellite	Date	U. T.	α δ Hirose's Method	α δ Yamazaki's Method	$\Delta\alpha(H-Y)$ $\Delta\delta(H-Y)$
1	Echo 2	'64, 7/22	$12^h41^m00^s.000$	$14^h29^m18^s.601$ $+20°53'$ $1''.60$	$14^h29^m18^s.588$ $+20°53'$ $1''.53$	$+0^s.013$ $+0''.07$
2	Echo 2	'64, 7/22	5.302	14 27 29.189 +22 16 22.27	14 27 29.165 +22 16 22.15	+0.024 +0.02
3	Echo 2	'64, 7/23	12 14 47.837	15 24 8.501 +25 26 6.98	15 24 8.476 +25 26 7.04	+0.025 −0.06
4	Echo 2	'64, 7/23	45.836	15 24 35.318 +24 46 18.95	15 24 35.302 +24 46 18.58	+0.016 +0.37
5	Echo 2	'65, 9/29	18 27 51.142	6 25 19.680 +27 21 22.70	6 25 19.670 +27 21 22.46	+0.010 +0.24
6	Echo 2	'65, 9/29	51.124	6 25 19.879 +27 21 21.96	6 25 19.869 +27 21 41.72	+0.010 +0.24
7	Echo 2	'65, 10/4	19 18 34.102	1 11 42.683 +18 35 40.31	1 11 42.690 +18 35 40.12	−0.007 +0.19
8	Echo 2	'65, 10/4	32.322	1 10 54.122 +18 54 07.89	1 10 54.125 +18 54 07.74	−0.003 +0.15

In this table, the third column contains the observed date, the fourth column contains the given times which we intend to ask the probable positions. The probable positions of satellite at these fimes computed by the method of Hirose and that of the author are contained in the fifth and sixth columns, respectively. The differences between them are shown in the seventh column.

Though the number of samples is small, the results by the two methods agree well with each other beyond expectation.

8. Systematic errors in measurement of the satellite trail images

Because of high angular velocity of satellites, equatorial mounting camera produces a trailed image for the satellite on photographic plate, while images of stars are kept stationary. In the measurement of positions of the trailed image, accordingly, we must consider a systematic error which has never appeared in the conventional photographic astrometry.

For a bright satellite, such as Echo, width of the trailed image reaches from 100 microns to 150 microns by using high speed emulsion. On the other hand, the dimensions of background star images are about 30~40 microns. Then, it seems to be possible that there is a systematic discrepancy in the relation between the positions of the stars regarded as the centre of dots and that of the satellite regrded as the centre of the trailed image. The following investigation has been tried by the author to find out this discrepancy.

The camera used through this investigations was a equatorial camera of 13 cm in aperture with focal length of 60 cm and was equipped with a new timing device developed by Ono (1966).

This timing device has a slit plate which traveles with a speed faster than that of the satellite image across a photographic plate during the passage of satellite through the field of the camera. The slit plate has several narrow slits of 0. 2 mm in width each. The exposures for satellite are given by these slits during very short time. As a result, a series of dot images of satellite are produced on the photographic plate, while the trailed image of the same satellite is recorded on the same photographic plate before and after the slits passage. Since the photographic situation of dot images and the star images may be regarded as same, the deviations of the dot images from the traild image seem to mean systematic discrepancy between the satellite and the background stars.

From this stand point, observations for Echo 2 were carried out and some succesful plate were obtained, while the plates on which the trailed image showed a large zigzag motion due to atmospheric irregularity were excluded. By using the second polynomial, the subdivided points on the trailed images were smoothed, and probable satellite trails were derived. Then the vertical distances from the dot images to the smoothed trail were calculated. These results are given in Table 3. Here, it should be remarked that during the passage of the slit plate to the satellite the trailed image is chopped by the slit plate ——in observations of this time, the passing times were about 1～2 seconds——, so the corresponding missing part of the trail was interpolated from trailed images in both sides.

In the fifth column of Table 3, *a*, *b* and *c* denote the dot images chopped by the three slits, respectively. These results clearly show a possibility of existence of systematic discrepancy between the satellite and the background stars, although values in the fifth column may involve some zigzag motions of long period caused by atmospheric irregularity and jitter in tracking.

TABLE 3. SYSTEMATIC DEVIATION BETWEEN DOTS' AND TRAILED IMAGES OF SATELLITE

Satellite	Date	Obs. Time	Images	Deviations (second of arc)
Echo 2	1965, 10/4	19^h18^m	*a*	−5″.2
			b	−3.8
			c	−3.1
Echo 2	1966, 3/16	11 40	*a*	+2.4
			b	+1.4
			c	+2.4
Echo 2	1966, 3/21	10 30	*a*	+2.8
			b	+2.1
			c	+4.8

Though these data are too poor to derive the definite conclusion on the cause of such systematic discrepancy at presnt, the following two causes seem to be plausible. One is the magnitude effect, well known to astronomers, arising from magnitude differences between satellite and stars, and the other is the error caused by measuring the centre of the ambiguous by broad image of trail. The author considers that the latter effect is rather effective than the former partly due to the bad quality of objective lens used.

Under-exposure during satellite passage may be a conceivable process to avoid these effects. However, because of the substantial differences of the shapes between the trailed image of satellite and the dot images of stars, its merit would not be so expected. Rather it may be better to exclude the errors by using a sufficient number of observations, because there are some evidences that the above systematic discrepances are at random in their directions and dimensions from plate to plate.

9. Comparison of two methods

As we have seen in the foregoing section, the trailing method has several advantages in comparison with the simultaneous method, but it also has some disadvantages. The following table provides the relative advantages of the both methods.

Trailing Method	Simultaneous Method
1. Only one station need a timing	1. All stations needs precise timing

device.

2. Necessary accuracy of timing is only 1/15 sec for accuracy 1″ in celestial coordinates.
3. In plate reductions, planetary aberration effect is not so serious.
4. Resulting accuracy depends on not only slant range to satellite, but also the geometrical relations of the base stations to the satellite's track.
5. For positioning an unknown station, three simultaneous observations at two known station and the unknown station are needed at least. Therefore, this method is not suitable to determine absolute positions.

devices.

2. Timing accuracy better than 1 msec is demanded for the apparent angular velocity of over 1000″/s.
3. Planetary aberration effect is serious.
4. Resulting accuracy is independent of the geometrical relations of stations to the satellite's track.
5. Two (or one) simultaneous observations at a known station and the unknown station, and one (or two) simultaneous observation at the unknown station and an another station, of which distance to the above known station are given, are needed, at least. By assuming only the distance between two stations, it is posible to determine absolute positions.

Although the trailing method needs satellites having adequate orbital inclination, it seems to be appropriate for determining geodetic positions of off-shore islands and secluded inland spots of poor facility of transportation because of simplicity and handiness of the equipments and economy of this observing system.

(Astronomical Section)

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