ON THE GEOMAGNETIC POST-PERTURBATION

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Abstract

The character of the Geomagnetic Post-Perturbation, shows a decreasing in the daily means of the magnetic horizontal intensity at many observatories during several days following magnetic storm, is illustrated in this paper. As the general tendency, the amplitude of the Post-Perturbation decrease proportionally with latitude except near the auroral zone. Also, the decrease value of Post-Perturbation on the stations of same longitude shows rather good agreement with each other. From these results, it can be assumed that the current system of Post-Perturbation, is determined from several observatories in low and middle latitude except high latitude observatories, is similar to equatorial ring current system. Therefore the radius of equatorial ring current system by Post-Perturbation able to be determined from world wide distribution of magnetic intensity of Post-Perturbation. Analysing much data in the worldwide, the results was obtained that the radius is about 20,000 km. It was found that this radius is equivalent to that of Van Allen Belts.

1. Introduction

Hitherto there are some investigations on the Post-Perturbation of the earth's magnetic field, which were studied by W. Van Bemmelen (1897), A. G. Mc Nish (1936), and E. H. Vestine (1947).

W. Van Bemmelen found a variation what be the "Post-Perturbation" or after disturbance effect. The variation is that after a storm the horizontal force become generally below the average and recovers after several days. He systematically studied the changes, from day to day, in the daily means of the three magnetic elements at many observatories.

E. H. Vestine reported the results of most detailed investigations on the characteristic properties of the Post-Perturbation. By means of his investigations, it was recovered that the Post-Perturbation effect, is reduced from the daily means minus monthly means, in most latitude is pronounced decrease in geomagnetic north component during several days following magnetic storm throughout the region from middle latitude to equator, and in geomagnetic east component the changes are relatively smaller in all latitude, also in vertical component the departures become smaller in lower latitude.

Recently the author reexamined in detail this disturbance in stations from middle latitude to low latitude through the world. The stations used

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here are listed in Table 1. For the systematic study of world-wide Geomagnetic Post-Perturbation it is necessary to examine the simultaneous records from many magnetic stations on the earth. But the variation of Post-Perturbation at a station near the auroral zone become irregular by the current system of auroral zone. Therefore the stations near the auroral zone is not included in this discussion.

TABLE 1. LIST OF MAGNETIC OBSERVATORIES IN THE MIDDLE AND LOWER LATITUDES.

Observatory	ϕ	λ	Φ	Λ
Cheltenham	38°.7N	283°.2E	50°.1	350°.5
Chambön	48.0N	2.3E	51.0	84.0
Tucson	32.2N	249.2E	40.4	312.2
San Juan	18.4N	293.9E	29.9	3.2
Kakioka	36.2N	140.2E	26.0	206.0
Tamanrasset	22.8N	5.5E	25.8	82.0
Honolulu	21.3N	201.9E	21.1	266.5
M' Bour	14.5N	18·5W	21.0	52.0

Notes: ϕ, λ = Geographic latitude and longitude
Φ, Λ = Geomagnetic latitude and longitude

2. Observational Results

The data collected for the purpose of studing the character of Post-Perturbation is records from 1949 to 1953. The daily mean change of the magnetic force from day to day is illustrated in Fig. 1 for four stations. Where, the value in figure is shown in geomagnetic component X', Y', Z transforming D, H, Z.

As shown in Fig. 1, the decreasing of the horizontal component below the average continue during several days following magnetic storm. This phenomenon is called the Geomagnetic Post-Perturbation. Such change is the non- cyclic changes. The general geographical distribution of the Post-Perturbation should be investigated from data over the earth in the individual components of the magnetic intensity, X' (geomagnetic northward), Y' (geomagnetic eastward), and Z (radially inward) in geomagnetic coordinate separately.

3. Longitude Effect of Post-Perturbation

By E. H. Vestine the decrease value of horizontal component of Post-Perturbation on stations of the same longitude exhibit rather good agreement with each other, except near the auroral zone, where considerable irregularity appears.

The average longitudinal distribution of the daily means minus monthly

Fig. 1 Running daily mean values on each component ~~five~~ four stations

March, 1950

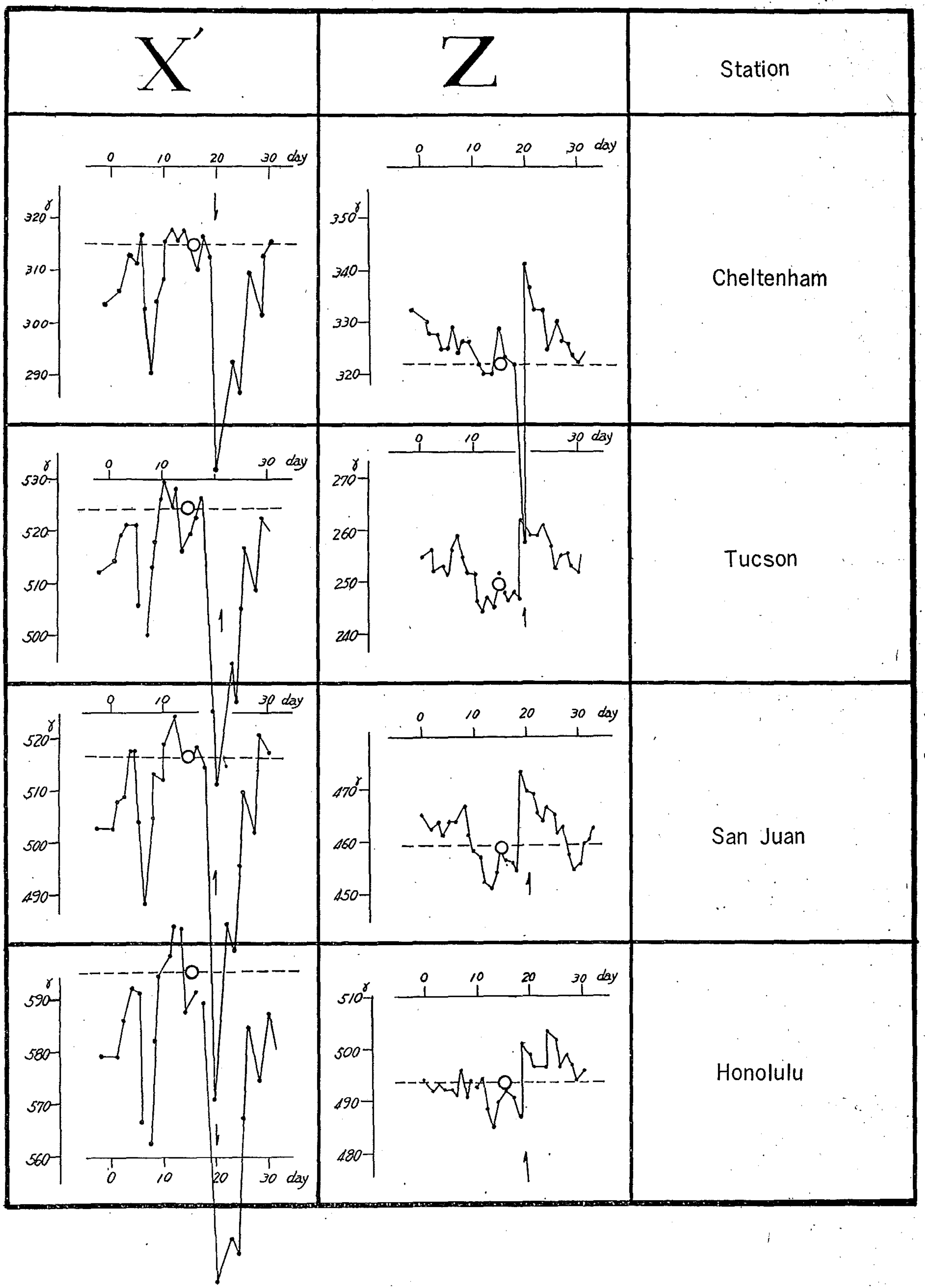


Fig. 2–1 Longitudinal effect on Post Perturbation

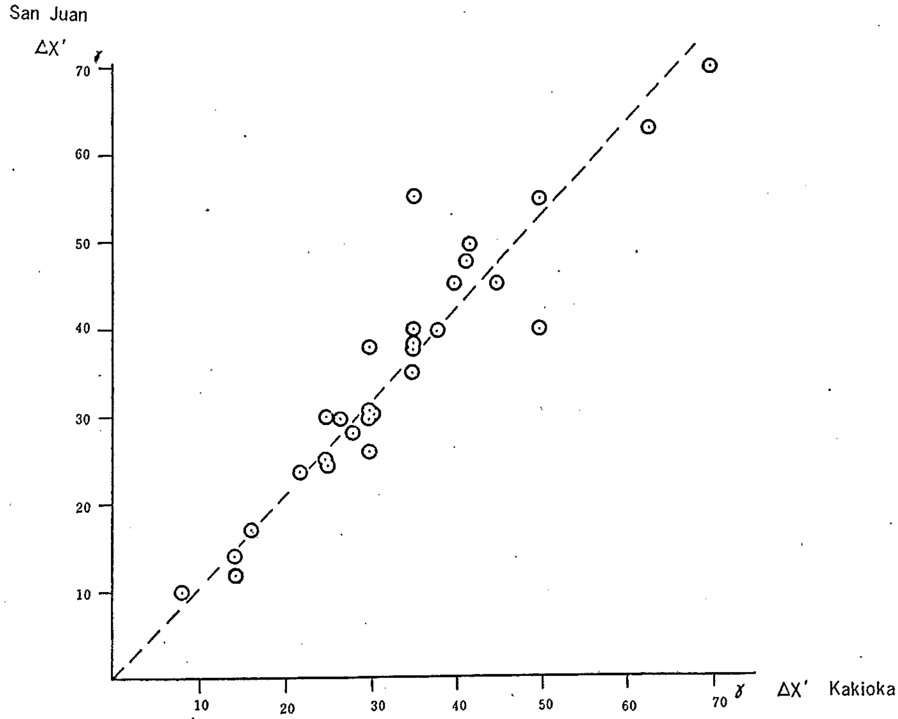


Fig. 2-2 Longitudinal effect on Post Perturbation

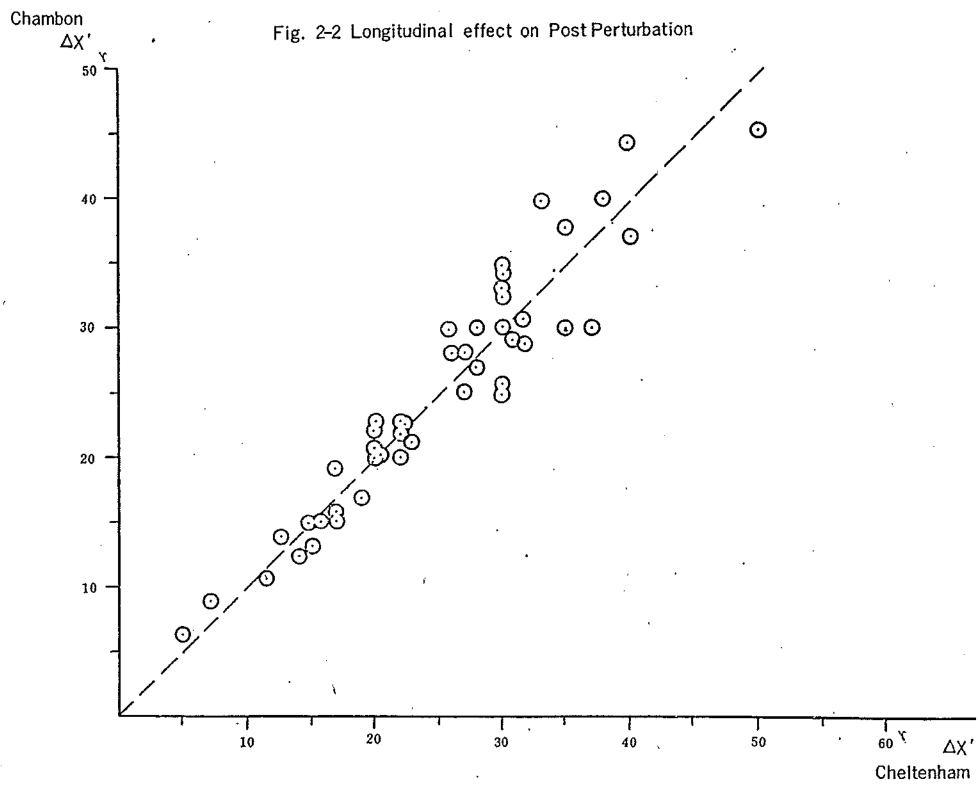


Fig.-3-1 Relation between two Station post perturbation ΔX'

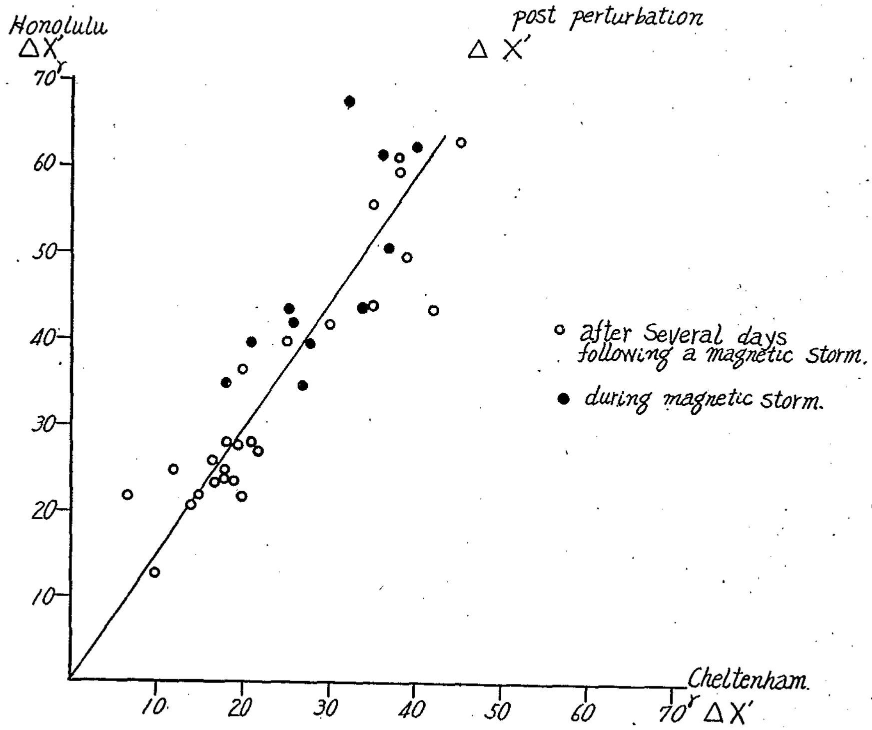
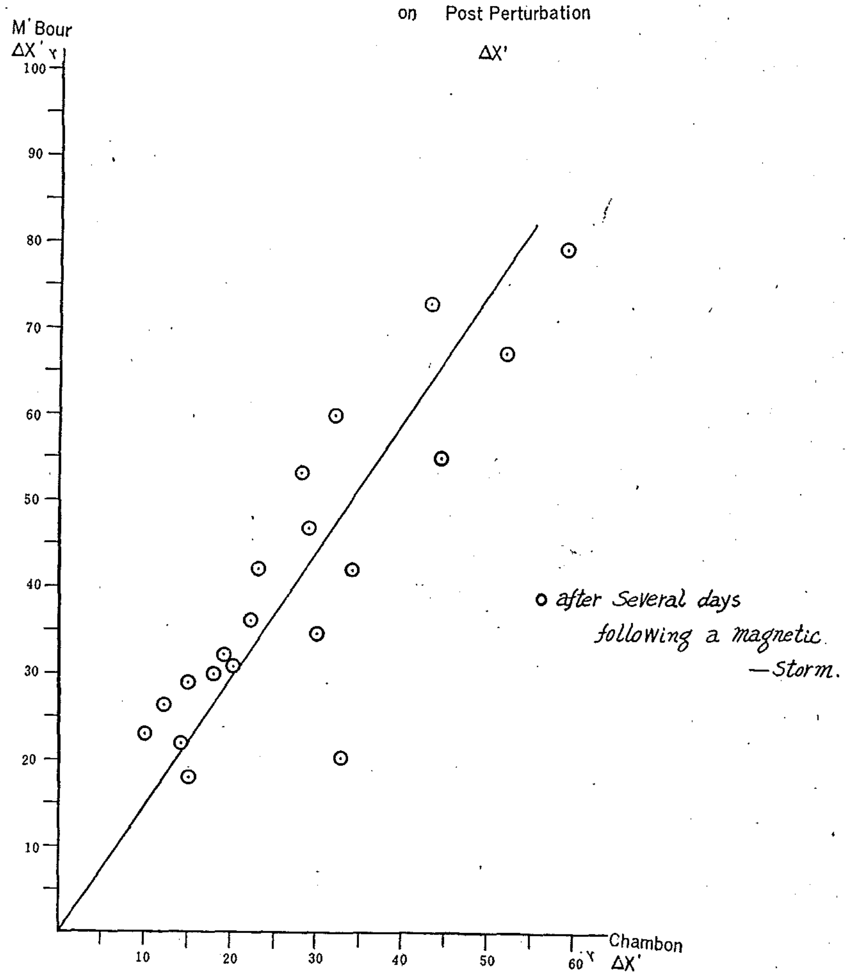


Fig. 3-2 Relation between two stations on Post Perturbation ΔX'



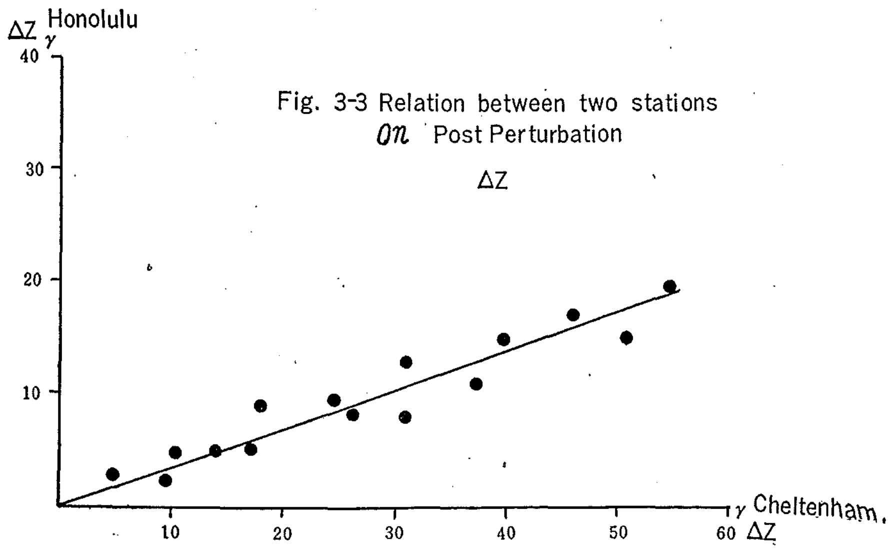


Fig. 3-3 Relation between two stations On Post Perturbation ΔZ

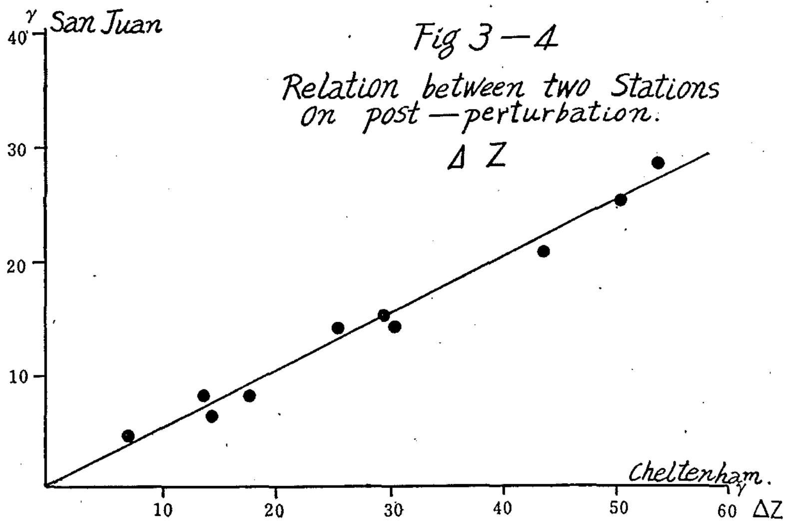
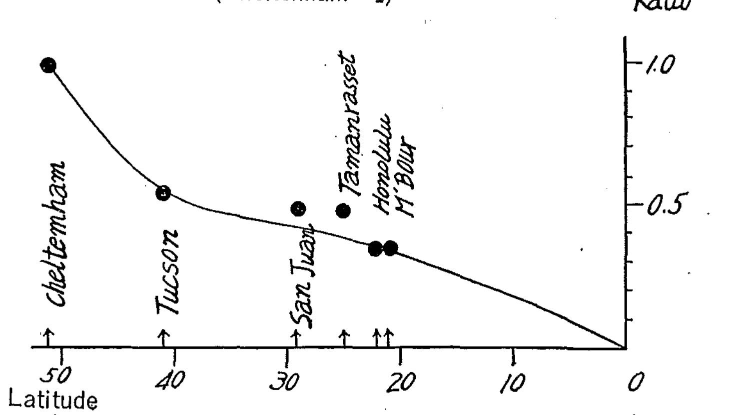


Fig 3—4 Relation between two Stations On post—perturbation. Δ Z

Fig. 4-1 Latitude distribution on Post Perturbation ΔZ (Cheltenham=1)



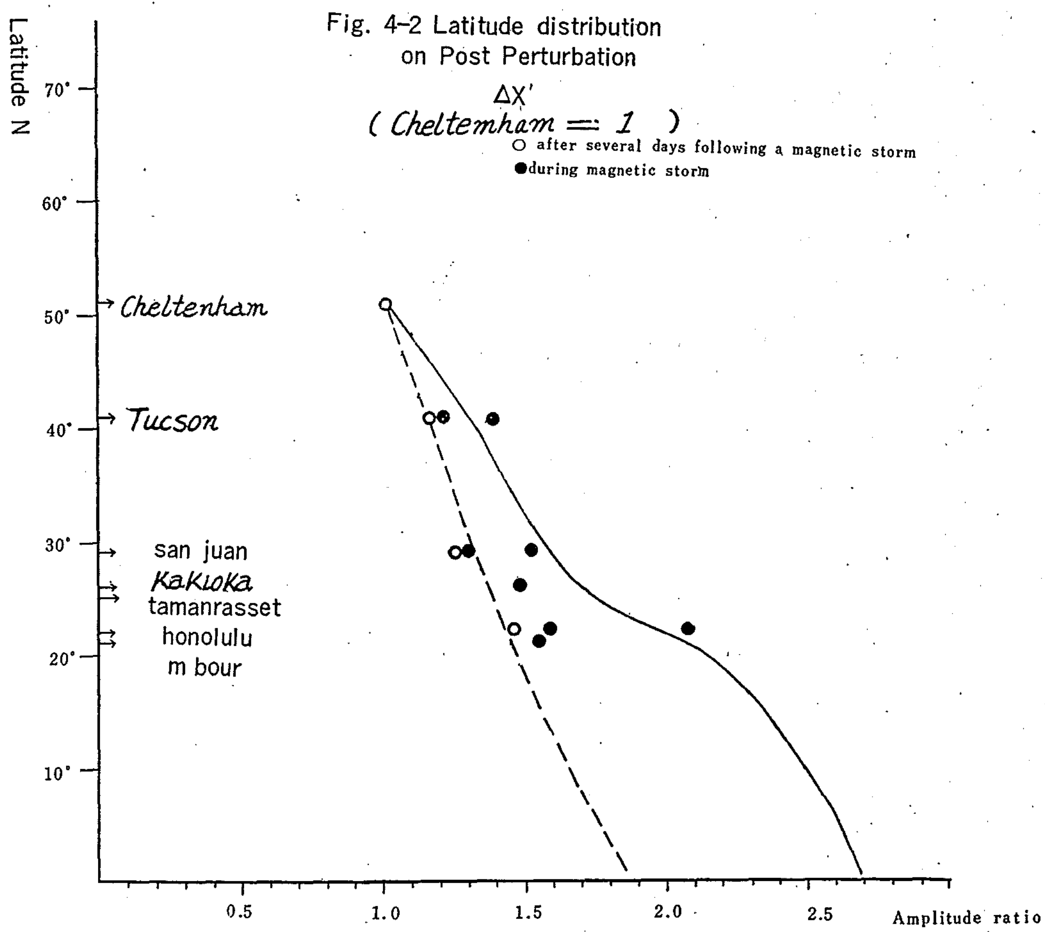
means on each components able to be obtained by plotting Fig. 2. As shown in Fig. 2, the decrease value of Post-Perturbation in low and middle latitudes is independent of the longitude of the station on each components.

4. Latitude Distribution of Post-Perturbation

It is reported by E. H. Vestine that the amplitude of north component of Post-Perturbation becomes smaller with increasing latitude.

At first, the author compared the amplitude of Post-Perturbation in two stations. The amplitude of Post-Perturbation in two stations is illustrated in Fig. 3.

In Fig. 3, the ordinate shows the changing value of Post-Perturbation at stations on three component (X', Y', Z). Then the latitude distribution of the amplitude ratio of Post-Perturbation observed at two stations in geomagnetic coordinate is shown in Fig. 4. As shown in Fig. 4, there is a tendency that amplitude decrease respectively with geomagnetic latitude on the geomagnetic north component and that the amplitude increases on the vertical component Z. In geomagnetic east component Y', the changes of Post-Perturbation are relatively smaller in all latitude.



5. Current System of Post-Perturbation

As shown above chapter, the field intensity of Post-Perturbation no influence on longtitude, and only vary with latitude. Then the field is influenced only depend on universal time.

In the first place, in order to investigate the current system of Post-Perturbation, the values, observed on the ground, included both the external and the internal parts of the Post-Perturbation field. Therefore, the separation of the magnetic field into external and the internal parts have to be done. The analysis used here is similar to those used by T. Rikitake(1950).

A magnetic potential V over the earth's surface can usually be expressed in terms of the series in polar coordinate,

$$V=\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left\{\left(e^m_{n,a}\frac{r^n}{R^{n-1}}+i^m_{n,a}\frac{R^{n+2}}{r^{n+1}}\right)\cos m\lambda+\left(e^m_{n,b}\frac{r^n}{R^{n+1}}+i^m_{n,b}\frac{R^{n+2}}{r^{n+1}}\right)\sin m\lambda\right\}P^m_n(\cos\theta)$$

where R is the radius of the earth, r the distance from the earth's center, θ the colatitude, and λ the east longitude.

As is well known, $e^m_{n,s}$ and $i^m_{n,s}$ are respectively the coefficients of the harmonics whose origin exist outside and within the earth, P^m_n $(\cos\theta)$ represent the parts of the spherical harmonic term. The north, east, and vertical intensities are

$X=\dfrac{1}{r}\dfrac{\partial V}{\partial\theta}$, $Y=-\dfrac{1}{r\sin\theta}\dfrac{\partial V}{\partial\lambda}$, and $Z=\dfrac{\partial V}{\partial r}$ respectively, then the components of the magnetic field on the earth's surface able to be expressed as follows.

North-component:

$$X=\sum_n\sum_m n\{(e^m_{n,a}+i^m_{n,a})\cos m\lambda+(e^m_{a,b}+i^m_{n,b})\sin m\lambda\}X^m_n$$

East-component:

$$Y=\sum_n\sum_m n\{-(e^m_{n,a}+i^m_{n,a})\sin m\lambda+(e^m_{n,b}+i^m_{n,b})\cos m\lambda\}Y^m_n \tag{1}$$

Downward:

$$Z=\sum_n\sum_m\{(ne^m_{n,a}-\overline{n+1}\,i^m_{n,a})\cos m\lambda+(ne^m_{n,b}-\overline{n+1}\,i^m_{n,b})\sin m\lambda\}P^m_n$$

where $X^m_n=\dfrac{1}{n}\dfrac{dP^m_n}{d\theta}$, $Y^m_n=\dfrac{m}{n\sin\theta}P^m_n$, as defined by Schmidt.

On the other hand, by means of spherical harmonic analysis,

$$\begin{aligned}
&\text{North component:} & X'&=\sum_n\sum_m(a^m_n\cos m\lambda+b^m_n\sin m\lambda)X^m_n\\
&\text{East component :} & Y'&=\sum_n\sum_m(a^{m\prime}_n\cos m\lambda+b^{m\prime}_n\sin m\lambda)Y^m_n\\
&\text{Downward〃 :} & Z&=\sum_n\sum_m(\tilde a^m_n\cos m\lambda+\tilde b^m_n\sin m\lambda)P^m_n
\end{aligned}\tag{2}$$

Equating the corresponding term of X and Y in (1) and (2),

$$n(e^m_{n,a}+i^m_{n,a})=a^m_n, \qquad n(e^m_{n,b}+i^m_{n,b})=b^m_n,$$

$$ne^m_{n,a}-\overline{n+1}\,i^m_{n,a}=\tilde{a}^m_n, \qquad ne^m_{n,b}-\overline{n+1}\,i^m_{n,a}=\tilde{b}^m_n$$

Solving this we obtain

$$e^m_{n,a}=\frac{(n+1)a^m_n+n\tilde{a}^m_n}{n(2n+1)}, \qquad i^m_{n,a}=\frac{a^m_n-\tilde{a}^m_n}{2n+1}, \text{ etc.} \tag{3}$$

The separation of the external and internal parts of the magnetic field able to be calculated from equation (3). In practice, getting the coefficients a^m_n, b^m_n, $\tilde{a}^m_n$ and $\tilde{b}^m_n$ from observed value of X' and Z by means of spherical harmonic analysis, then the coefficient of magnetic field induced by means of external current system is gotten.

The results practically is given from observed value using the longitude and the latitude distribution of the Post-Perturbation field that is illustrated in Fig. 2 and Fig. 4. Now, by means of above results that the field intensity of Post-Perturbation is independent on longitude, we can consider as following.

The current system of Post-Perturbation that is determined from several observatories in low and middle latitude except high latitude observatories, are expected to have magnetic field like of a equatorial ring current system. Therefore, if it able to be assumed that the current system of Post-Perturbation is equivalent with a equatorial ring current of radius a, the radius of current system of Post-Perturbation able to be determined from observed value.

Meanwhile, the magnetic field X and Z by a equatorial ring current at a point on the earth's surface as Fig. 5 can be expressed in the following terms of the series by Nagaoka(1921).

$$X=\frac{\pi}{a^3}xy\left\{3+\frac{45}{8}\frac{x^2}{a^2}-\frac{15}{2}\frac{y^2}{a^2}+\frac{525}{64}\frac{x^4}{a^4}-\frac{525}{16}\frac{x^2y^2}{a^4}+\frac{105}{8}\frac{y^4}{a^4}+\cdots\cdots\right\}$$

$$Y=\frac{\pi}{a}\left\{2+\frac{3}{2}\frac{x^2}{a^2}-3\frac{y^2}{a^2}+\frac{45}{32}\frac{x^4}{a^4}-\frac{45}{4}\frac{x^2y^2}{a^4}+\frac{15}{4}\frac{y^4}{a^4}+\frac{6175}{128}\frac{x^6}{a^6}+\cdots\cdots\right\}$$

The direction of the magnetic force proceed outward from the (x, y) plane, as shown in Fig. 5, and the direction of the magnetic force by means of a equatorial ring current at a point on the earth's surface incline from geomagnetic earth axis. The inclination varies by means of the latitude of stations. The inclination is calculated from the following equation.

$$\tan\Delta\theta=\frac{X}{Y}$$

The inclination on the earth surface with regard to the equatorial ring current of various radius is illustrated in Fig. 6.

On the other hand, the inclination $\Delta\theta'$ from geomagnetic axis by the external field of Post-Perturbation observed on the earth surface can be gotten

Fig. 5 Magnetic field on the earth surface induced by a ring current

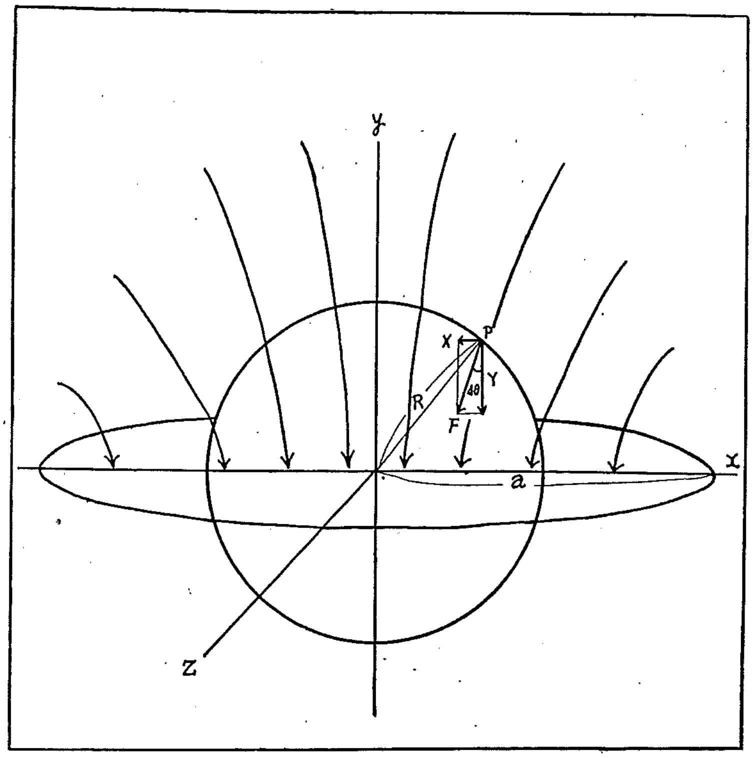
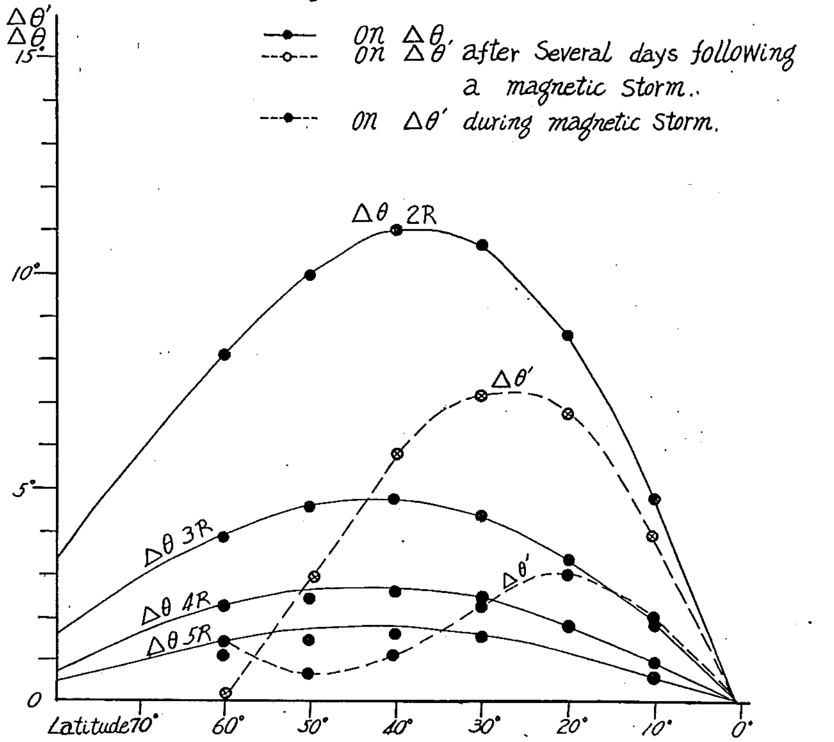
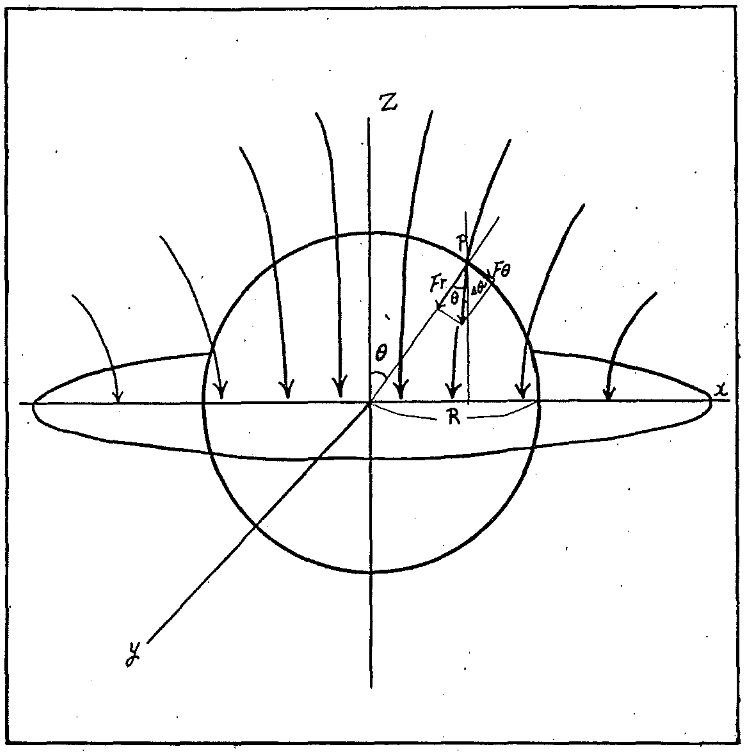


Fig. 6 Inclination on the earth surface Δθ with regard to the equatorial ring current of various radius and Δθ' with regard to magnetic field of post—perturbation.



by means of the same method, that is, $\Delta\theta'$ is calculated from following equation and Fig. 7.

Fig. 7 Magnetic field on the eafth surfaceinduced by the Post Perturbation



$$\tan\theta' = \frac{X'}{Z} \qquad \text{or} = \frac{F\theta}{F_r}$$

$$\Delta\theta' = \theta - \theta'$$

Where,

θ : colatitude

$\Delta\theta'$: inclination

X' : geomagnetic north component of external field

From such analysis, Latitude distribution of $\Delta\theta'$ shown in Fig. 6 is gotten. By comparing $\Delta\theta$ and $\Delta\theta'$ in Fig. 6, we can determine the most probable radius of the current system of Post-Perturbation, if it is assumed that the current system of Post-Perturbation is an equatorial ring currents.

6. Conclusions

In the present study, the cause of the Geomagnetic Post-Perturbation is not clear, but from above results, the possibility of the presentation of such equatorial ring current induced on the surface of a cavity of outer atmosphere is suggested. The most probable radius of the current system of Post-Perturbation is at the distance of 3~4 earth radii on several days after magnetic storm, and become nearly the distance of 2 earth radii during magnetic storm.

According to the recent investigation of outer atmosphere by Pioneer, the existence of high energy particles region, Van Allen Belts, in the earth's outer atmosphere was suggested. The outer zone is at the distance of 3~4 earth radii. Comparing with the result of investigation by Van Allen and the one of theoretically expected by the present investigation about Post-Perturbation field, it was found that the agreement is generally good. Therefore it may be concluded that the current system of Post-Perturbation is induced in the Van Allen Belts.

7. Acknowledgment

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References

Chapman S. 1951, Geomagnetism **1**, **2**.
Nagaoka H. 1921, Phil. Mag. **41**, 377-388.
Rikitake T. 1950, Bull. Earthq. Res. Inst. **28**, 46.
Slaucitajs L. and Mc Nish A. G. 1936, Int. Union Geod. Geophys. **10**, 289-301.
Van Bemmelen W. 1897, Terr. Mag. **2**, 74-76.
Vestine E. H. 1947, The Geomagnetic Field, It's Description and Analysis. Carnegie Institution of Washington Publication 580